Monitoring Groundwater Quality using Probability Distribution in Gwale, Kano state, Nigeria

Auwalu Ibrahim[1], Ahmad Abubakar Suleiman [1], Usman Aliyu Abdullahi[1], Suleiman Abubakar Suleiman[2]

[1]*Department of Statistics, Kano University of Science and Technology, Wudil 713281, Nigeria*
[2]*Department of Hydrology, Kano State Ministry of Water Resources, Dr. Bala Muhammad Road 700222, Kano State – Nigeria*

Corresponding Author: aiamaigora1@gmail.com

Abstract

Groundwater is the water present beneath the earth's surface in soil pore spaces and the fractures of rock formations. Establishing a probability distribution that provides a good fit to groundwater quality has recently become a topic of interest in the fields of hydrology, meteorology among others. In this paper, three groundwater datasets including calcium, magnesium, and chloride are fitted to the normal, lognormal, gamma, Weibull, logistic, and log-logistic distributions to select the best groundwater model. The measures of goodness of fits such as the Akaike information criterion (AIC), Bayesian information criterion (BIC), and log-likelihood are computed to compare the fitted models. The results show that the gamma distribution gives better fits for calcium and magnesium datasets while the lognormal distribution provides a better fit for the chloride dataset than other competing models. This research describes an application of probability distributions and the best-fitted distribution to a practical problem involving groundwater data analysis. By assuming the distribution of data, analysts can utilize the characteristics of the distribution to make predictions on outcomes.

Keywords: Gamma Distribution, Groundwater Quality Data, Lognormal Distribution, Maximum Likelihood, Probability Distribution

1. Introduction

Groundwater is one of the abundant and reliable sources of drinking water found in natural environments and is vulnerable to deterioration caused by domestic, industrial, and agricultural activities (Suleiman et al., 2020a). The deterioration in the quality of drinking water has a negative health effect for humans, plants, and animals; which may introduce water-borne diseases such as kidney disease, gastroenteritis, maternal and infant mortality, cholera, hypertension, prostate, and colorectal cancer, miscarriage, birth defects in children typhoid fever and giardiasis (Arabi et al., 2013; Taiwo et al., 2015; Chang et al., 2018). The World Health Organization (WHO) reported that groundwater contamination is responsible for 1.7 million infant deaths annually (WHO, 2017). Hence, sustainable management of available water resources becomes a periodic need in arid and semi-arid regions.

In developing countries, rapid population growth coupled with the rate of urbanization and economic development tends to impair the groundwater resources and result in high variability for many water quality parameters. The possible variability may be due to anthropogenic activity and natural variance during different seasons through biochemical or chemical processes (Mustapha et al., 2014). In the last decades, there has been a gradual deterioration in the quality of purity observed in groundwater because of so many human activities such as the rapid population growth, agricultural activities, urbanization, and industrialization, which has exposed groundwater resource to the risk of contamination (Adewoyin et al., 2019). Today urban regions in Nigeria exhibit a high level of dependence on groundwater for urban water-supply, notably for innumerable domestic and industrial activities (Tukur et al., 2018), and the situation has resulted in further pollution of groundwater sources (Emenike et al., 2019). Various treatment processes exit to reduce drinking water pollution, such as managed aquifer recharge, activated carbon treatment, ozonation and so on (Kiefer et al., 2020).

In recent times, few methods such as the projection pursuit technique, neural networks, chemical analysis, and water quality index have been considered as the most reliable way to obtain information about the quality of water (Salman and Ruka'h 1999; Sadat-Noori et al., 2014). Also, several statistical methods such as multivariate statistical techniques, analysis of variance and frequency analysis have been used to monitor the quality of drinking water (Maryam et al., 2018; Gulgundi and Shetty 2018; El Baghdadi et al., 2019; Garba et al., 2017; Mustapha, 2014). Mathematical models are often used as decision support tools to evaluate contamination in groundwater (Cecilia et al., 2020).

Furthermore, the probability distribution model is one of the recent statistical tools applied in hydrology for water estimation and prediction. This arises from experiments where the outcome cannot be predicted with certainty (Patel and Shete 2012). Groundwater variables are subject to the uncertainty that is assumed to have a particular probability distribution model. The probability distribution model helps to analyze and interpret spatial and temporal variations of hydro-morphological as well as physical and chemical parameters of groundwater easily and can serve as a reference model for future investigations (Kishore et al., 2011). Commonly used probability distributions known as normal, lognormal, gamma, Weibull, and log-logistic distributions have been applied to the analysis of rainfall and surface water datasets (Nwaiwu et al., 2005; Maryam et al., 2018; Karim and Hossein 2019). Recently, several probabilistic risk assessment models were to evaluate groundwater quality (Roshni et al., 2020; Opoku et al., 2020; Suleiman et al., 2020a).

The previous studies mainly focused on applications of probability distribution models for predicting annual rainfall and estimating the probable return period of the rainfall datasets. However, these studies did not account for relative probability distributions that will likely provide better statistical estimation for the dataset. Hence, this paper aims to determine suitable probability distribution models for calcium, magnesium and chloride ions collected from Gwale local government area, northwestern Nigeria. High or low intake of these variables can lead to irregular heartbeat, cardiovascular disease, anxiety, insomnia, nervousness, weakness, muscle/joint pain, osteoporosis, epilepsy, high stomach acid, asthma, high blood pressure, PMS, anxiety, sweating, muscle spasms/cramps, dysmenorrhea, angina, constipation, migraine/headaches, cardiovascular disease, arrhythmia, cardiac arrest, coma, muscle spasms, joint/spinal degeneration, bone loss, low stomach acid, low body temperature, low blood pressure, increased risk of multiple cancers, bowel / genitourinary bleeding, dry skin, fatigue, depression, vomiting, diarrhea (Arabi et al., 2013). The AIC, BIC, and log-likelihood statistics are used to ascertain the best-fit probability models for collected groundwater quality datasets. The determined models could serve as efficient models for monitoring groundwater quality especially in predicting the quality characteristics of water parameters for future investigations.

2. Materials and methods

2.1. Study area

Kano has been known as the most populous city in Nigeria and the largest administrative state in the northern region of Nigeria (World Bank Group, 2016). Kano metropolis covers an area of 600km^2 situated in the northwestern part of Nigeria, located between latitude 10˚ and 12˚N and longitude 8˚ and 9˚E (Amoo et al., 2018; Suleiman et al., 2020b). Politically Gwale Local Government Area falls under 8 metropolitan local government areas, boarded with Dala to the North, Kumbotso to the South, Kano Municipal to the East, and Ungogo to the West. Gwale has many industries situated heavily at Sharada Phase III. The study area map is shown in Fig. 1.

The present climate of Kano is the tropical wet and dry type with a dry season between 4 – 5 months and wet season May and September (Dan Azumi and Bichi, 2010; Suleiman et al., 2020b). There are two major geological structures in the Kano region, with minor intrusion as the third. The larger are of the south and north. West is indentation by rocks of basement complex with the intrusion of younger granite in the extreme southern parts. To the north-west area are the unconsolidated sediments of the Chad republic. These two structures are separated by a transitional zone which constitutes the well define hydro-geological divide of the region. Two hydrological can be identified in the region. The rivers are located in the upland areas, which comprise river Kano and river Challawa. The area received rainfall of over 800mm annually. The temperature varies by warm to hot seasons between November and February. Annual mean temperature ranges from about 21 degrees Celsius to 27 degrees Celsius (Bala et al., 2011; Suleiman et al., 2020a).

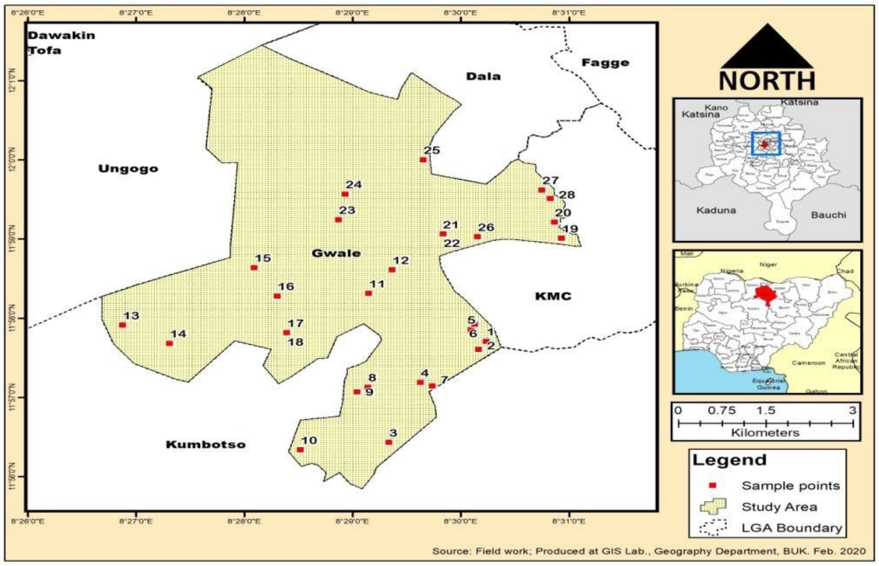

Figure 1. The map of Gwale area showing sampling locations

2.2. Sample selection

Groundwater data were collected from 28 locations in Gwale area, northwestern Nigeria in August 2018. The area has a population of 362059 at the 2006 census. The locations were randomly selected using simple random sampling to ensure an equal chance of representativeness. The latitude and longitude of the chosen locations were determined on a map using a geographical position system. The groundwater samples were obtained from hand pumps and open wells which were stored in iced plastic containers before taken to the laboratory according to the standard method (Standard Methods, 2005). In this research, the chosen water parameters are calcium, magnesium, and chloride ions. The concentrations of these ions expressed in milligram per liter were analyzed in the federal ministry of water resources laboratory, Kano state, Nigeria.

2.3. Probability distribution function (pdf) of groundwater samples

Any experiments whose outcomes cannot be exactly predicted with certainty are termed as uncertain. An event like water quality concentration is uncertain (Loucks and Van 2017), which can be adequately predicted by developing suitable probability distribution models. For predictive purposes, it is important to determine the appropriate probability distribution of the underlying distribution of data by understanding its parameters (Chaibandita and Konyaib 2012). The parameters of the individual distribution such as location, shape, and scale are necessary to describe the distribution. These parameters will also allow the distribution to provide flexibility and effectiveness in modeling situations (Surendran and Tota-Mahara

2015). The location parameter simply shifts the graph to left or right on the horizontal axis. The scale parameter allows distribution to take on a variety of shapes depending on the value of the shape parameter. The scale parameter also describes the stretching capacity of the probability distribution. In general, a change in the location parameter will shift the distribution; a change in the scale parameter will stretch or shrink the distribution. The probability distributions used in this paper are briefly defined in the following equations:

2.3.1. Normal distribution

The pdf of the normal distribution is defined by:

$$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; \quad -\infty \leq x \leq \infty \tag{1}$$

where x is the groundwater data ranging from $-\infty$ and $+\infty$, μ is the mean value and σ is the standard deviation.

2.3.2. Lognormal distribution

A random variable x is said to have a lognormal distribution with two parameters μ and σ if its pdf is given by:

$$f(x)=\frac{1}{\sqrt{2\pi x^2\sigma^2}}e^{-\frac{1}{2}\left(\frac{lnx-\mu}{\sigma}\right)^2} \quad ; \quad 0 \leq x \leq \infty . \tag{2}$$

where x is the groundwater data ranging from 0 and $+\infty$, the parameters μ and σ are the mean and standard deviation of the distribution respectively.

2.3.3. Gamma distribution

The pdf of this distribution is given by:

$$f(x)=\frac{1}{\Gamma(\alpha)\varphi^{\alpha}}x^{\alpha-1}e^{-\frac{x}{\varphi}} \quad ; \quad 0 \leq x \leq \infty . \tag{3}$$

where x is the groundwater data ranging from 0 and $+\infty$, φ is the scale parameter and α is the shape parameter.

2.3.4. Weibull distribution

Weibull distribution has its probability distribution function expressed by:

$$f(x)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \quad ; \quad 0 \le x \le \infty . \tag{4}$$

where x is the groundwater data ranging from 0 and $+\infty$, α is the shape parameter and β is the scale parameter.

2.3.5. Logistic distribution

A random variable x has the logistic distribution with two parameters μ and λ if its pdf is given by

$$f(x)=\frac{e^{-(x-\mu)/\lambda}}{\lambda\left(1+e^{-(x-\mu)/\lambda}\right)^{2}} \quad ; \quad -\infty \le x \le \infty . \tag{5}$$

where μ and λ are the location and scale parameters of the logistic distribution respectively.

2.3.6. Log-logistic distribution

A random variable x is said to follow a log-logistic distribution with two parameters α and β if its probability distribution function is given by:

$$f(x)=\frac{\left(\beta/\alpha\right)\left(x/\alpha\right)^{\beta-1}}{\left(1+\left(x/\alpha\right)^{\beta}\right)^{2}} \quad ; \quad 0 \le x \le \infty . \tag{6}$$

where α and β are the scale and shape parameters of the log-logistic distribution respectively.

2.4. The goodness-of-fit

This is a method of choosing one from among a set of competing models. There are several approaches to determine the optimal model for a given dataset. Some of these approaches are AIC, BIC, and Log-likelihood. These approaches are used to assess the relative quality of the various statistical models, and details can be found in (Sasireka et al., 2019; Laio et al., 2009).

For independent samples, the maximum likelihood estimates is given by:

$$L\left(\theta; x_1,\ldots,x_n\right)=f\left(x_1,\ldots,x_n \mid \theta\right)=\prod_{i=1}^{n} f\left(x_i \mid \theta\right) \tag{7}$$

The logarithm of the likelihood function is usually used as the estimator. The unknown parameters, denoted by the vector θ in the above equation, are calculated by maximizing the log-likelihood function.

$$\log(L) = \log\left(\sum_{i=1}^{n} f(x_i \mid \theta)\right) \tag{8}$$

The AIC is obtained as:

$$AIC = -2\log(L) + 2k \tag{9}$$

where k is the number of model parameters and $\log(L)$ is the log-likelihood function for the statistical model. The BIC is expressed as:

$$BIC = \log(n)k - 2\log(L) \tag{10}$$

where n is the data sample size. To select the best model, three statistical approaches namely, the AIC, BIC, and log-likelihood are used to assess how well the chosen models fit the groundwater datasets. In general, it can be selected as the best model the one which has the smaller values of the AIC, BIC statistics, and the larger values of the log-likelihood (Gündüz and Korkmaz 2020). Also, histogram plots are given to support the numerical results. The probability models of this study could serve as efficient models for monitoring groundwater quality especially in predicting the quality characteristics of water parameters for future investigations. It should be noticed that all results in this work have been obtained under the maximum likelihood estimation method and all computations are carried out using R software.

3. Results and discussion

A total of six probability models such as normal, lognormal, gamma, Weibull, logistic and log-logistic models are used to fit groundwater datasets (calcium, magnesium, and chloride).

The maximum likelihood estimates (MLEs) and their corresponding standard errors (in parentheses) of the model parameters, and the AIC, BIC, and log-likelihood values for the calcium, magnesium, and chloride datasets obtained for normal, lognormal, gamma, Weibull, logistic and log-logistic models are reported in Table 1, 2 and 3, respectively. The results in Table 1 show that only the gamma distribution gives an adequate fit while the normal, lognormal, Weibull, logistic and log-logistic distributions do not give an adequate fit for the calcium dataset. Also, it is evident from Table 2 that the fitted gamma model corresponds to the lowest values of the AIC and BIC, and the largest value of the log-likelihood (for magnesium dataset) among the fitted normal, lognormal, Weibull, logistic and log-logistic models. Therefore, the gamma model can be chosen as the best model for the magnesium dataset. For the chloride dataset in Table 3, the lognormal model provides the best fit.

The histogram plots of calcium, magnesium, and chloride datasets and the estimated probability distribution functions of the competitive models are displayed in Fig. 2, 3, and 4, respectively. It is clear from Tables 1, 2, and 3, and Fig. 2, 3, and 4 that the gamma and lognormal models provide better fits to these three groundwater datasets.

Table 1. The MLEs and their standard errors (in parentheses), and the statistics AIC, BIC, and log-likelihood for the calcium dataset

Models	MLEs (shape)	MLEs (scale)	AIC	BIC	Log-likelihood
Normal	σ = 81.52143 (8.06561)	μ =42.67921 (5.70325)	293.6684	296.3328	-144.8342
Lognormal	σ =4.25626 (0.10491)	μ =0.55515 (0.07418)	288.8543	291.5187	-142.4272
Gamma	φ =3.61686 (0.92360)	α =0.04437 (0.01215)	288.3897	291.0542	-142.1949
Weibull	2.04221 (0.29738)	92.44501 (9.05035)	289.2745	291.9389	-142.6372
Logistic	σ =77.61928 (8.24623)	μ =24.73370 (3.85030)	294.6912	297.3556	-145.3456
Log-logistic	β =3.06445 (0.47193)	α =71.38920 (7.79406)	290.2434	292.9078	-143.1217

Table 2. The MLEs and their standard errors (in parentheses), and the statistics AIC, BIC, and log-likelihood for the magnesium dataset

Model	MLEs (scale)	MLEs (shape)	AIC	BIC	Log-likelihood
Normal	12.35463 (1.22189)	6.46565 (0.86401)	187.9847	190.6491	-91.99237
Lognormal	2.36956 (0.10486)	0.55488 (0.07415)	183.1712	185.8356	-89.58562
Gamma	3.61952 (0.92619)	0.29291 (0.08040)	182.7085	185.3729	-89.35426
Weibull	2.04323 (0.29751)	14.00982 (1.37086)	183.5949	186.2593	-89.79743
Logistic	11.76578 (1.24948)	3.74785 (0.58355)	189.0055	191.6699	-92.50275
Log-logistic	3.06501 (0.47198)	10.81724 (1.18063)	184.5632	187.2276	-90.28162

Table 3. The MLEs and their standard errors (in parentheses), and the statistics AIC, BIC and log-likelihood for the chloride dataset

Model	MLEs (scale)	MLEs (shape)	AIC	BIC	Log-likelihood
Normal	134.28571 (14.84198)	78.58928 (10.50194)	327.8577	330.5221	-161.9289
Lognormal	4.72487 (0.11309)	0.59844 (0.07997)	319.3008	321.9652	-157.6504
Gamma	3.01163 (0.26845)	0.02242 (0.00614)	320.2588	322.9232	-158.1294
Weibull	1.82960 (0.26845)	152.17260 (16.65456)	321.4998	324.1642	-158.7499
Logistic	125.74447 (15.70753)	46.64032 (7.24190)	329.7430	332.4074	-162.8715
Log-logistic	2.738051 (0.41545)	111.36334 (13.79706)	321.9131	324.5775	-158.9566

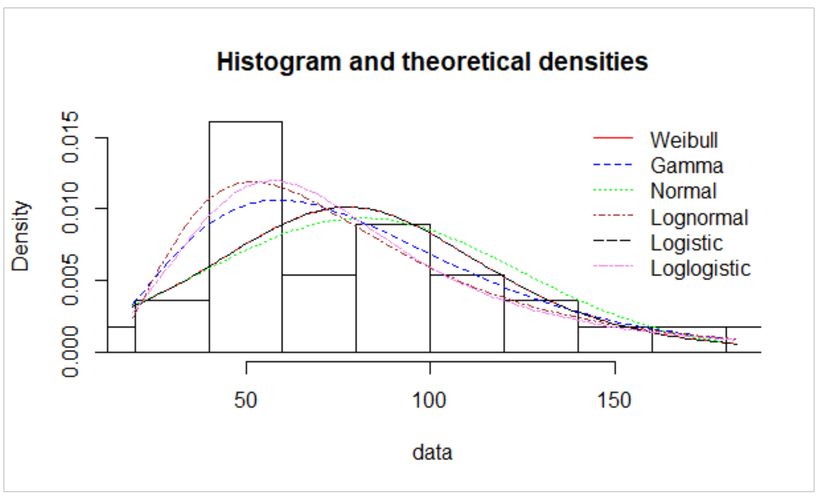

Figure 1: Plots of the fitted distributions for calcium dataset

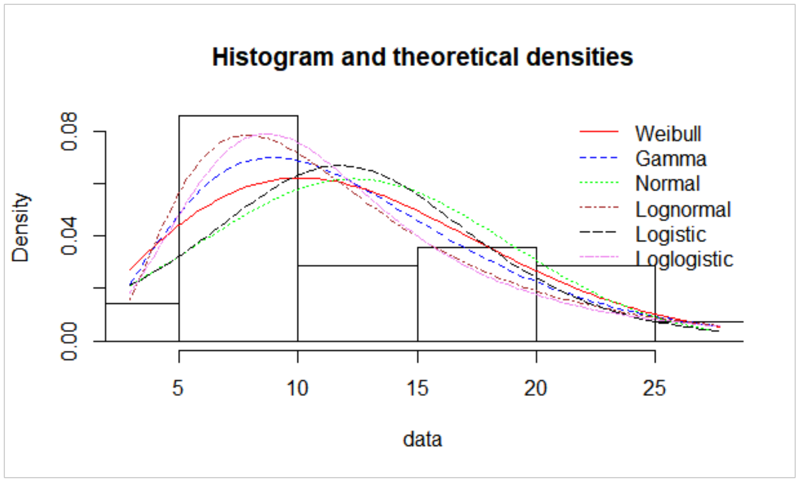

Figure 2: Plots of the fitted distributions for magnesium dataset

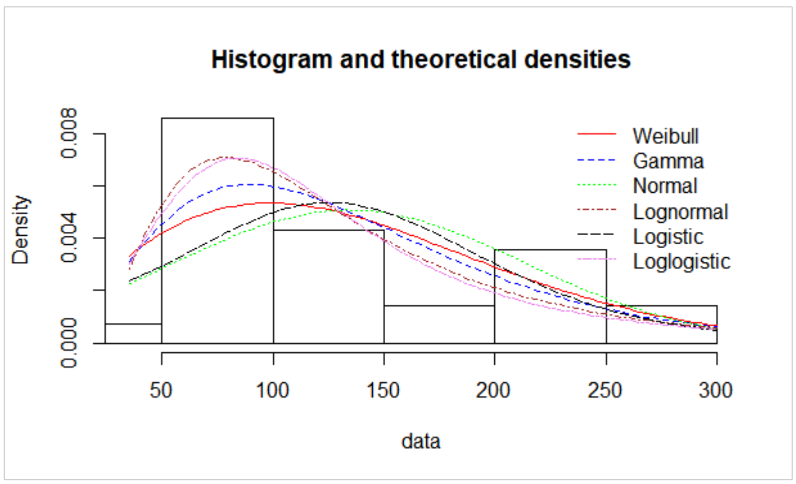

Figure 3: Plots of the fitted distributions for chloride dataset

4. Conclusions

There has been an increased interest in monitoring groundwater quality based on statistical techniques. The applications of probability distributions in modeling hydrological data have attracted the attention of several researchers to monitor the probabilistic behavior in water datasets. In this paper, three groundwater datasets such as calcium, magnesium, and chloride were fitted to the normal, lognormal, gamma, Weibull, logistic and log-logistic distributions to select the best groundwater model. The AIC, BIC, and log-likelihood statistics are used to ascertain the best-fit probability models for collected groundwater quality datasets. The results show that the gamma distribution gives better fits for calcium and magnesium datasets and lognormal distribution provides a better fit for the chloride dataset than other competing models. This research describes an application of probability distributions and the best-fitted distribution to a practical problem involving groundwater data analysis. By assuming the distribution of data, analysts can utilize the characteristics of the distribution to make predictions on groundwater variables.

5. Acknowledgments

The project was financially supported by the Tertiary Education Trust Fund (TETFund) under the Institutional Based Research (IBR) grant allocated to Kano University of Science and Technology, Wudil, Nigeria in 2018. Also, the authors would like to offer sincere thanks to the editorial team and the referee of this journal for their valuable contributions in this paper.

6. References

Adewoyin, O.O., Kayode, O.T., Omeje, O., & Odetunmibi, O.A. (2019). Risk Assessment of Heavy Metal and Trace Elements Contamination in Groundwater in Some Parts of Ogun State. *Cogent Engineering*. 6:1, 1632555, https://doi.org/10.1080/23311916.2019.1632555.

Amoo, A., Adeniyi A. & Hamza, Y.G. (2018). Assessment of Groundwater Quality in Sharada Industrial Area of Kano, North-Western Nigeria. *FUW Trends in Science & Technology Journal*. 3(2A), 407 – 411.

Arabi, A.S., Funtua, I.I, Dewu, B.B.M., Garba, M.L., Okoh, S., Kwaya, M.Y., & Bolori, M.T. (2013). Assessment of Calcium and Magnesium Concentrations in Groundwater as Supplements for Sleep Related Ailments. *J. Appl. Environ. Biol. Sci*. 3(7), 29-35.

Bala, A.E., Eduvie O.M., & Olly, B. (2011). Borehole Depth and Regolith Aquifer Hydraulic Characteristics of Bedrock Types in Kano Area, Northern Nigeria. *African Journal of Environmental Science and Technology*. 5(3).

Chaibandita, K., & Konyaib, S. (2012). Using Statistics in Hydrology for Analyzing the Discharge of Yom River. *APCBEE Procedia* 1, 356 – 362.

Chang, K.Y., Wu, I.W., Huang, B.R., Juang, J.G., Wu, J.C., Chang, S.W., & Chang, C.C. (2018). Association Between Water Quality Measures and Chronic Kidney Disease Prevalence in Taiwan. *Int J Environ Res Public Health*. 15(12), 2726, doi:10.3390/ijerph15122726.

Cecilia, L.D., Porta, G.M., Tang, F.H.M., Riva, M., & Maggi, F. (2020). Probabilistic Indicators for Soil and Groundwater Contamination Risk Assessment. *Ecological Indicators*. 115, 106424.

Dan Azumi, S., & Bichi, M.H. (2010). Industrial Pollution and Heavy Metals Profile of Challawa River in Kano. *Nig. J. Appl. Sci. Envt. Sanitation*. 5(1), 23-29.

El Baghdadi, M., Medah, R., & Jouider, A. (2019). Using Statistical Analysis to Assess Urban Groundwater in Beni Mellal City (Morocco). *International Journal of Agronomy,* 2019. Article ID 7469741, 22 pages. https://doi.org/10.1155/2019/7469741

Emenike, P.C., Tenebe, I., Ogarekpe, N., Omole, D., & Nnaji, C. (2019). Probabilistic Risk Assessment and Spatial Distribution of Potentially Toxic Elements in Groundwater Sources in Southwestern Nigeria. *Scientific Reports*. 9, 15920, https://doi.org/10.1038/s41598-019-52325-z.

Garba, A., Ekanem, E.O., Garba, H.I., & Mustapha, A. (2017). Chemometric Application on the Physico-Chemical Assessment of Groundwater (Borehole) from Hadejia Local Government Area of Jigawa State, Nigeria. *J. Chem. Soc. Nigeria*. 42(1), 42-48.

Gündüz, S., & Korkmaz, M.C. (2020). A New Unit Distribution Based on the Unbounded Johnson Distribution Rule: The Unit Johnson Su Distribution. *Pakistan Journal of Statistics and Operation Research*. 16(3), 471-490, http://dx.doi.org/10.18187/pjsor.v16i3.3421.

Gulgundi, M. S., & Shetty, A. (2018). Groundwater Quality Assessment of Urban Bengaluru Using Multivariate Statistical Techniques. *Applied Water Science*. 8, 43, https://doi.org/10.1007/s13201-018-0684-z.

Karim, H.M., & Hossein, S. (2019). Determination of the Best Fit Probability Distribution for Annual Rainfall in Karkheh River at Iran. *World Academy of Science, Engineering and Technology. International Journal of Environmental and Ecological Engineering*. 13(2), 69-75.

Kiefer, K, Bader, T., Minas, N., Salhi E., Janssen, E.M.-L., Gunten, U.V., & Hollender, J. (2020). Chlorothalonil Transformation Products in Drinking Water Resources: Widespread and challenging to Abate. *Water Research*. 183, 116066, https://doi.org/10.1016/j.watres.2020.116066.

Kishore, K.D., Bhanita, D., Bhupen, K.B., & Abani, K.M. (2011). Development of New Probability Model with Application in Drinking Water Quality Data. *Advances in Applied Science Research*. 2(4), 306-313.

Laio, F., Baldassarre, G.D., & Montanari, A. (2009). Model Selection Techniques for the Frequency Analysis of Hydrological Extremes. *Water Resources Research*. 45, W07416, doi:10.1029/2007WR006666.

Loucks, D.P., & Van, B.E. (2017). An Introduction to Probability, Statistics and Uncertainty in Water Resources Systems Planning and Management. *Springer,Cham*. ISBN 978-3-319-44232-7,doi:10.100/978-3-319-44234-1-6.

Maryam, G., Kaveh, O.A., Saed, E., & Vijay, P.S. (2018). Analyzing the Groundwater Quality Parameters Using Frequency Analysis. *American Journal of Engineering and Applied Sciences*. 11(2), 482-490, doi.org/10.3844/ajeassp.2018.482.490

Mustapha, A., Aris, A.Z., Yusoff, F.M., Zakaria, M.P., Ramli, M.F., Abdullah, A.M.,· Kura, N.U., & Narany, T.S. (2014). Statistical Approach in Determining the Spatial Changes of Surface Water Quality at the Upper Course of Kano River, Nigeria. *Water Qual Expo Health*. 6, 127–142, doi: 10.1007/s12403-014-0117-7.

Nwaiwu, N.E., & Apagu, B. (2005). Fitting Probability Distributions to Component Water Quality Data From a Treatment Plant. *Global Journal of Environmental Sciences*. 4(2), 151-154.

Opoku, P.A., Anornu, G.K., Gibrilla, A., Owusu, E.J., Ganyaglo, S.Y., & Egbi, C. D. (2020). Spatial Distribution and Probabilistic Risk Assessment of Exposure to Heavy Metals in Groundwater in a Peri-Urban Settlement: Case Study of Atonsu-Kumasi, Ghana Groundwater for Sustainable development, 10: 100327, doi.org/10.1016/j.gsd.2019.100327.

Patel, N.R., & Shete, D.T. (2012). Probability Distribution Analysis of Consecutive Days Rainfall Data for Sabarkantha District of North Gujarat Region, India. *ISH Journal of Hydraulic Engineering*. 14(3), 43-55, doi.org/10.1080/09715010.2008.10514921.

Roshni, T., Choudhary, S., Jha, M.K., & Mandal, N. (2020). *Probability-Based Approach for Evaluating Groundwater Risk Assessment in Sina Basin, India*. Publisher: Elsevier, Chapter 12, Handbook of Probabilistic Models, ISBN 9780128165140, pp. 289-304. Doi.org/10.1016/B978-0-12-816514-0.00012-6.

Standard Methods for the Examination of Water and Wastewater. (2005). *The 21st edn, American Public Health Association/American Water Works Association/water Environment Federation, Washington DC, USA*.

Sadat-Noori, S.M., Ebrahimi, K., & Liaghat, A.M. (2014). Groundwater Quality Assessment Using the Water Quality Index and GIS in Saveh- Nobaran Aquifer, Iran. *Environ Earth Sci*. 71(9), 3827–3843.

Salman, S.R., & Ruka'h Y.A. (1999). Multivariate and Principal Component Statistical Analysis of Contamination in Urban and Agricultural Soils From North Jordan. *Environ Geol*. 38(3), 265–270.

Sasireka, K., Suribabu, C.R., & Neelakantan T.R. (2019). Extreme Rainfall Return Periods Using Gumbel and Gamma Distribution. *International Journal of Recent Technology and Engineering*. 8, 2277-3878, doi.10.35940/ijrte.D1007.1284S219.

Suleiman, A.A., Ibrahim, A., & Abdullahi, U.A. (2020a). Assessment of Probability Distributions of Groundwater Quality Data in Gwale Area, North-Western Nigeria. *Annals of Optimization Theory and Practice*. 3(1), 37-46, doi: 10.22121/aotp.2020.243381.1039.

Suleiman, A.A., Ibrahim, A., & Abdullahi, U.A. (2020b). Statistical Explanatory Assessment of Groundwater Quality in Gwale LGA, Kano state, Northwest Nigeria. *Hydrospatial Analysis*. 4(1), 1-13, doi: org/10.21523/gcj3.2020040101.

Surendran, S. & Tota-Maharaj, K. (2015). Log logistic Distribution to Model Water Demand Data. *Procedia Engineering*. 119, 798 – 802

Taiwo, A.M., Towolawi, A.T., Olanigan, A.A., Olujimi, O.O., & Arowolo, T.A.(2015). Comparative Assessment of Groundwater Quality in Rural and Urban Areas of Nigeria, *Research and Practices in Water Quality*, IntechOpen,*, pp.179 – 191.* http://dx.doi.org/10.5772/59669.

Tukur, A. I., Nabegu, A. B., Umar, D. A., Olofin., E. A., & Sulaiman, W.A. (2018). Groundwater Condition and Management in Kano Region, Northwestern Nigeria. *Hydrology*. 5, 16, doi.10.3390/hydrology5010016.

World Health Organization, 2017. The cost of a polluted environment, *http://www.who.int/mediacentre/news/releases/2017 /pollution-child-death/en/.*

World Bank Group, 2016. From oil to cities: Nigeria's next transformation. *World Bank Group: Directions in Development, Washington, DC: World Bank*. doi: 10.1596/978-1-0792-3.