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The Use of Standardized Exponentiated Gumbel Error Innovation Distribution to Forecast Volatility: A Comparative Study

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RESEARCH ARTICLE

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Abstract

This study is designed to model several selected volatility models using a newly developed error innovation distribution called Standardized Exponentiated Gumbel Error Innovation Distribution (SEGEID) to determine the efficiency and effectiveness of the model in terms of its adaptability and forecast evaluation. SEGEID improves some existing error distributions and uses the standard&Poor-500 index data returned from 2004 to 2022.The use of this error innovation distribution, GJR-GARCH (1,1), has been shown to be more effective than other volatility models considered in this study. The results of the study show that GJR-GARCH (1,1) is better than GARCH (1,1), EGARCH (1,1) and TGARCH (1, 1) because it has the lowest AIC(-17.0394) and RMSE (0.00011).

Keywords: Error Innovation, Exponentiated, Forecast, GARCH, Volatility

1. Introduction

One way to create wealth in a country is through financial investments such as the stock market (Nigeria Stock Exchange's return index, Standard and Poor 500 return index, crude oil price return index and many others). For example, stock investment has the main objective of achieving profits. In fact, stock yield measurement is the primary criterion for evaluating investments, not prices. Modelling and predicting volatility in financial markets has attracted much attention in recent years. Volatility is an indicator of unpredictability that can have a serious impact on risk management, investment decisions and even national monetary policy. Variance is a statistical measure of the fluctuation of its value around the average. Market volatility is an indicator of investor risk exposure. Error Innovation Distribution is one of the basic techniques to estimate parameters of any volatility model. Therefore, the research objective is to model some selected volatility models using the newly developed Standardized Exponential Gumbel Error Innovation Distribution (SEGEID) developed by Olayemi and Olubiyi (2022) to determine the efficiency and effectiveness of models in terms of their effectiveness and forecasting assessment

2. Theoretical Analysis

The volatility of stock returns is defined by Olowe (2009) as the variation of stock prices. Stock return volatility is not immediately observed because there is only one trading price per day, which is one of the main asset return structures. Although the phenomenon cannot be observed directly, it can still be studied because it has patterns similar to those of other phenomena. These patterns include clustering and leverage. Therefore, several researchers have given various models to model volatility. The model proposed (not limited) was the Auto-Regressive Conditional Heteroscedasticity (ARCH) model introduced by Engle (1982) to explain heteroscedasticity in data. Bollerslev (1986) proposed a new model, the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, as improvement to ARCH model. In order to address the deficiencies of the ARCH/GARCH model, Nelson (1991) created the Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) model. Taking into account the asymmetric relationships between returns and volatility, the EGARCH model was proposed as a solution to the shortcomings of the GARCH model, that is inability to model data that are not normally distributed. Given the nature of the effect of leverage more than an exponential rather than a quadratic, the E-GARCH model predicts algorithmic conditional variation. The definition of volatility in terms of the logical transformation is an important advantage of the E-GARCH model compared to the symmetrical GARCH model since it requires parameters to be limited to ensure positive variance (Jose, 2010).

Samson et al. (2020) employed skewed error innovation distribution to simulate volatility in the Nigerian stock market in order to discover the volatility model and skewed error distribution that best describe the dynamics in the volatility of the Nigerian stock market. Parameters of GARCH (1,1), APARCH (1,1), GJR-GRACH (1,1), IGARCH (1,1), and EGARCH (1,1) were estimated using skewed normal, skewed Student-t, and skewed generalized error distributions (1,1). The time range covered by the data utilized was from October 2, 2001, to March 29, 2018. The parameters of these volatility models were calculated for each error distribution using the RUGARCH function in R. Based on the results, the skewed normal distribution is the most advantageous error distribution, both in terms of fitness and in comparison, to the other error distributions used in the majority of the models. Based on the least RMSE, the APARCH (1,1) skewed normal distribution was suggested as the optimal forecasting model. The results show that the Nigerian stock market is very volatile and clustered, and that this volatility is highly persistent.

Ogenyi and Umeh (2019) studied inflation volatility in Nigeria from 1980 to 2010 by employing the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) methods. The analysis found that factors such as prior inflation, GDP, government expenditure, debt levels, import/export rates, and currency rates all contributed to inflation's unpredictability throughout the period under consideration. The unemployment and interest rates of the previous were years also shown to have significantly mitigated the volatility of Nigeria's inflation. In light of these results, the author recommended a number of measures, including better fiscal and monetary policy harmonization to keep prices stable, more spending on agricultural research so that farmers have access to cutting-edge technology, and improving the present market structure.

Ekong and Onye (2018) have determined the best model for predicting the returns of Nigeria's shares and the types of volatility associated with the returns of Nigeria's shares with daily data from all shares. In order to choose the most accurate model, this study estimates six groups of asymmetric and asymmetric GARCH-family stock-return volatility models (three of which are increased by the volume of transactions) for three groups of error distributions: normal, student t, and general error distribution (GED). The root mean square error (RMSE) and the inequality coefficient of Thiel were used to conclude that GED's GEDGARCH (1,1) and improved E-GARCH (1,1) had the best prediction performance compared to previous forecasts out of samples over30 days. Olayemi et al. (2022) Studies have been conducted to determine which model is best suited to address price fluctuations in the Nigerian crude oil market. In their analysis, they investigated the GARCH, EGARCH, and POWER conditional heteroscedasticity models (PARCH). Comparisons between GARCH (1, 1), E-GARCH (1,1) and PARCH (1, 1) have been made to predict the prices of Nigerian crude oil. Their information comes from the Database of the Central Bank of Nigeria and includes2,422 observations collected in 12 years (2010-2021). The effectiveness of the GARCH model is measured by using Akaike's information criteria.

3. Methodology

Computation of return series for price. Let

$$r_{sk} = log\left(\frac{y_t}{y_{t-1}}\right) t = 1,2,\dots,n \tag{1}$$

where y_t and y_{t-1} are the present and previous closing prices at time t and r_{sk} is the returns series.

3.1 Stock Market Volatility

Generally, in the financial market, volatility is often known as standard deviation σ or variance σ^2. The volatility of a stock is a gauge of the degree of uncertainty surrounding the returns it will produce. The parameter is often generated using a number of data from the empirical sample in the manner shown below:

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \mu)^2 \tag{2}$$

where μ is the mean return and $r_{sk} = log\left(\frac{p_t}{p_{t-1}}\right)$.

3.2 Computation of the Conditional Error Term

The Error (ε_t) term is computed as:

$$\varepsilon_t = r_t - \mu \tag{3}$$

where r_{sk} is the return of the series and μ is the mean of the series. For single observation return series, the error term is given as:

$$\varepsilon_i = r_i - \mu \tag{4}$$

where ε_i is the individual error term, r_i is the individual return series and μ is the grand mean of the whole return series.

3.3 Computation of the Variance Term

The unconditional variance computation formula is given as:

$$\sigma_t^2 = var(r_{sk}) \tag{5}$$

where r_{sk} is the return of the series. For single observation return series, the variance is given as:

$$\sigma_{t-1}^2 = var(r_1, r_2) \tag{6}$$

3.4 Volatility Models

3.4.1 Autoregressive Conditional Heteroscedasticity (ARCH) Model

Engle (1982) proposed the ARCH (q) model which formulates volatility model as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 \tag{7}$$

This can also be expressed as:

$$\sigma_t^2 = \alpha_0 + \sum_i^q \alpha_i \varepsilon_{t-i}^2 \tag{8}$$

where $\alpha_0 > 0, \alpha_i \geq 0; i = 1, \dots, q$ the parameters of the model and q is the order of ARCH terms.

3.4.2 Generalize Autoregressive Conditional Heteroscedasticity (GARCH) Model

The GARCH (p, q) model proposed by Bollerslev (1986) formulates volatility as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 \tag{9a}$$

Alternatively, it can be stated as:

$$\sigma_t^2 = \alpha_0 + \sum_i^q \alpha_i \varepsilon_{t-i}^2 + \sum_j^p \beta_j \sigma_{t-j}^2 \tag{9b}$$

where $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0 \; and \; \alpha_i + \beta_j < 1$ for all i and j while q is the ARCH order terms, and p is the GARCH order terms.

3.4.3 Exponential Generalised Autoregressive Conditional Heteroscedasticity (E-GARCH) Model

The E-GARCH (p, q) model was proposed by Nelson (1991) to formulate the volatility model as follows :

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^p \alpha_i \left[\lambda \varepsilon_{t-i} + \gamma \left\{ |\varepsilon_{t-i}| - \sqrt{\frac{2}{\pi}} \right\}\right] + \sum_{j=1}^p \beta_j \log(\sigma_{t-j}^2) \tag{10}$$

$\alpha_0 > 0, \alpha_i \geq 0, \gamma \geq 0, \beta_j \geq 0$ and $\alpha_i + \beta_j + \gamma/2 < 1$ are the parameters of the model.

3.4.4 Threshold Generalised Autoregressive Conditional Heteroscedasticity (TGARCH) Model

The Threshold GARCH model was proposed by Zakoian (1994) to formulate the volatility model as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \gamma_i S_{t-i} \varepsilon_{t-i}^2 \tag{11}$$

$\alpha_0 \geq 0, \alpha_1 \geq 0, \gamma \geq 0, \beta_j \geq 0$ where, S_{t-i} is an indicator for negative ε_{t-1} that is S_{t-i} is 1 if $\varepsilon_{t-1} < 0$ and 0 $otherwise$.

3.4.5 Glosten Jagannathan Runkle (GJR)- Generalised Autoregressive Conditional Heteroscedasticity (GJR-GARCH) Model

The GJR-GARCH model was proposed by Glosten et al., (1993) to formulate the volatility model as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q (\alpha_i + \gamma_i M_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \tag{12}$$

$\alpha_0 \geq 0, \alpha_1 \geq 0, \gamma \geq 0, \beta_j \geq 0$ where, M_{t-i} is an indicator for negative ε_{t-1} that is M_{t-i} is 1 if $\varepsilon_{t-1} < 0$ and 0 $otherwise$.

3.4.6 Standardized Exponentiated Gumbel Error Innovation Distribution (SEGEID)

The Standardized Exponentiated Gumbel Error Innovation Distribution (SEGEID) was proposed by Olayemi and Olubiyi (2022) to formulate the error innovation distribution. The distribution is as follows:

$$g\left(\varepsilon_t;\alpha,\sigma_t\right)=\frac{\alpha}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\left[1-\exp\left\{-\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right\}\right]^{\alpha-1}\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\exp\left\{-\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right\}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right) \quad (13)$$

α is the shape parameter, $\boldsymbol{\sigma_t}$ is the volatility models with vector parameters.

4. Results and Discussion

4.1 Empirical Result

The S&P 500 index returns were empirically analysed in this series. As shown in Table 1, the results show that the average returns are positive, negative and index returns are high. The results of the Jarque-Bera statistics show that the NSE index returns series are not normally distributed since the *p*-value is less than 1%.

Table 1: Descriptive statistic

Statistics	Returns of S&P-500 Index
Mean	1.000031
Std. Dev.	0.001550
Skewness	-0.433658
Kurtosis	14.86901
Jarque-Bera	40220.23
P-Value	0.001
Observations	4927

4.2 Normality Test

Here, we carried out further test (normality test) to reaffirm Jarque-Bera result. The results of the normalization test for the S&P-500 return are shown in Table 2. Further analysis using Kolmogorov–Smirnov (K-S) and Shapiro–Wilk (S-W) statistics showed that the returns series for NSE shares are not normally distributed because the *p*-values were below 0.01.

Table 2: Test of Normality of the Return of Standard and Poor 500 (S&P500) Index Return

	Kolmogorov-Smirnov (S-K)			Shapiro-Wilk (S-W)		
	Statistics	Df	*p*-value	Statistics	Df	*p*-value
S&P500	0.118	2610	0.000	0.875	2610	0.000

4.3 Stationarity Test

By observing the time patterns of the series, researchers were able to investigate the stability of the return series. The price and return series of the S&P-500 are stationary, as shown in Figure 1. There was also an official test of stationarity using the Dickey-Fuller (ADF) Augmented Test. The results show that the Dickey-Fuller extended test statistics are all lower than the critical values of 0.01 in Table 3, which means that all returns are stationary and there are no unit roots and transformation requirements.

Table 3: Augmented Dickey-Fuller (ADF) Test of Stationarity of All-Share Index Return Series Standard and Poor 500

Stocks		ADF Test Statistics	Comment
S&P-500 Index Returns	Intercept	-40.26943	Stationary at the stated level without transformation
	Trend and Intercept	-40.38089	Stationary at the stated level without transformation

1% critical = -3.342675

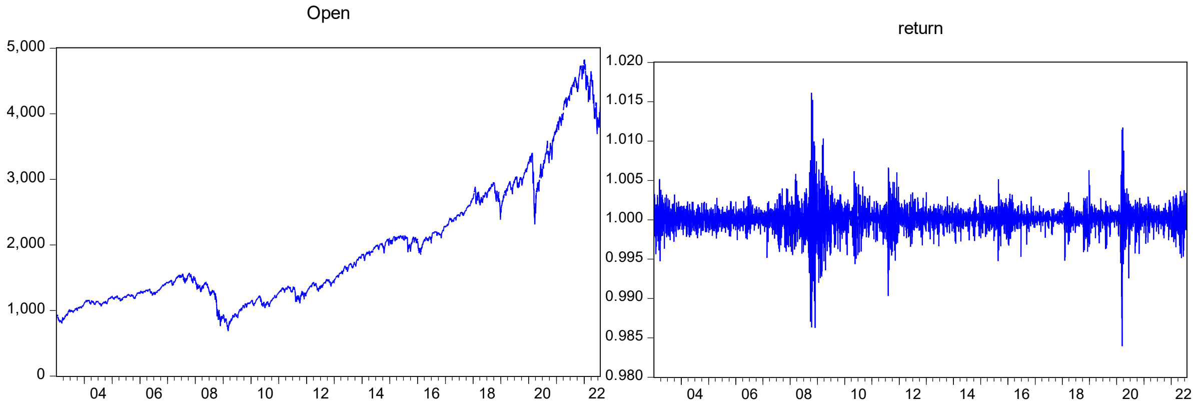

Figure 1: Volatility Plot of both Price and Returns of S&P-500 Index Returns

4.4 ARCH Effect Test

It was tested using Lagrange Multiplier (LM) methods. Table 4 summarizes the p-value and F statistics achieved at different delays. In the case of S&P-500 stock returns, the F-statistic value is significant, at 1%. Therefore, returns to the S&P-500 index meet the heteroscedasticity model requirements and provide evidence of the presence of the ARCH effect.

Table 4: Lagrange Multiplier Test of the Presence of ARCH Effect

	ARCH Effect	F-Statistic	p-value
S&P500 Index Returns	At lag 1-2	246.53	0.001
	At lag 1-5	175.41	0.001
	At lag 1-10	106.00	0.001

4.5 Estimates of the Parameters of some GARCH Family Models based on Standard and Poor-500 Index Returns

Tables 5 and 6 show the standardized exponentiated Gumbel error innovation distribution (SEGEID) based on the S&P500 index return data and maximum likelihood estimates, as well as the parameters estimates of the GARCH family model. The coefficient β_1 (a factor affecting longevity) is important in all models. Most models found that a small volatility change followed a similar small change, while a significant increase in volatility was accompanied by a similar large change($p < 0.05$ and $p < 0.01$). The γ_1 coefficient asymmetry and the statistical significance of positive and negative effects on the 5% and 10% levels of the GARCH family were found in all GARCH family models studied($p < 0.05$). The leverage effect is to test whether there is a negative relationship between asset returns and volatility

Table 5: Estimates of the Parameters of some GARCH Family Models on Standard and Poor-500 Index Returns Using the Six Existing and the Newly Developed Error Innovation Distributions.

Model	Error	w	α_1	β_1	γ_1	Shape
GARCH (1,1)	NORM	1.564×10^{-05}	$1.859\times10^{-01***}$	$8.530\times10^{-01***}$		
	STD-T	2.284×10^{-04}	$1.168\times10^{-01***}$	$8.760\times10^{-01***}$		5.016^{***}
	GED	1.764×10^{-05}	$1.947\times10^{-01***}$	$8.510\times10^{-08***}$		5.016^{***}
	SNORM	1.874×10^{-07}	$1.737\times10^{-01***}$	$8.720\times10^{-06***}$		5.016^{***}
	SSTD-T	1.684×10^{-05}	$1.747\times10^{-01***}$	$8.520\times10^{-07***}$		5.016^{***}
	SGED	1.864×10^{-06}	$1.738\times10^{-01***}$	$8.820\times10^{-07***}$		5.016^{***}
	SEGEID	0.15575	-0.24616	1.11573^{*}		8.5634
GJR-GARCH (1,1)	NORM	5.564×10^{-10}	$3.758\times10^{-01***}$	$1.001\times10^{-08***}$	0.7883^{**}	2.000^{***}
	STD-T	$7.786 \times 10^{-03*}$	$1.852\times10^{-01***}$	$7.964\times10^{-01***}$	-0.0773^{**}	
	GED	$2.082 \times 10^{-06***}$	$1.268\times10^{-01***}$	$6.524\times10^{-01***}$	0.4873^{**}	1.153^{***}
	SNORM	5.764×10^{-10}	$3.748\times10^{-01***}$	$1.001\times10^{-08***}$	-0.7683^{*}	
	SSTD-T	$7.785 \times 10^{-03*}$	$1.842\times10^{-01***}$	$5.664\times10^{-01***}$	-1.11553^{*}	
	SGED	$2.071 \times 10^{-06***}$	$1.369\times10^{-01***}$	$7.724\times10^{-01***}$	0.03703^{**}	1.154^{***}
	SEGEID	0.25066	0.0051	0.00018	-0.5963^{*}	7.5543

Table 6: Cont.'s on Estimates of the Parameters of some GARCH Family Models on Standard and Poor-500 Index Returns Using the Six Existing and the Newly Developed Error Innovation Distributions.

Model	Error	w	α_1	β_1	γ_1	Shape
EGARCH (1,1)	NORM	6.564×10^{-10}	$4.648\times10^{-01***}$	$1.01\times10^{-08***}$	0.5873^{**}	2.001^{***}
	STD-T	$8.785 \times 10^{-03*}$	$1.862\times10^{-01***}$	$6.754\times10^{-01***}$	-0.5603^{**}	
	GED	$3.081 \times 10^{-06***}$	$1.268\times10^{-01***}$	$7.524\times10^{-01***}$	0.3783^{**}	1.163^{***}
	SNORM	6.564×10^{-10}	$4.648\times10^{-01***}$	$1.01\times10^{-06***}$	-0.6873^{*}	2.001^{**}
	SSTD-T	$6.765 \times 10^{-03*}$	$1.862\times10^{-01***}$	$6.754\times10^{-01***}$	-1.11373^{*}	
	SGED	$1.081 \times 10^{-06***}$	$2.267\times10^{-01***}$	$7.524\times10^{-01***}$	0.01603^{**}	2.162^{***}
	SEGEID	0.26486	0.00561	0.00007	-0.76843^{*}	7.5643
TGARCH (1,1)	NORM	5.764×10^{-10}	$2.745\times10^{-01***}$	$1.00\times10^{-08**}$	0.7783^{**}	2.002^{***}
	STD-T	$6.765 \times 10^{-03*}$	$2.752\times10^{-01***}$	$7.754\times10^{-01***}$	-0.0873^{**}	
	GED	$3.081 \times 10^{-06***}$	$1.268\times10^{-01***}$	$7.524\times10^{-01***}$	0.3683^{**}	1.182^{***}
	SNORM	6.564×10^{-10}	$3.859\times10^{-01***}$	$1.01\times10^{-08***}$	-0.7883^{*}	
	SSTD-T	$7.866 \times 10^{-03**}$	$1.842\times10^{-01***}$	$6.954\times10^{-01***}$	-1.1185^{*}	
	SGED	$3.081 \times 10^{-05***}$	$1.268\times10^{-01***}$	$7.523\times10^{-01***}$	0.02603^{**}	1.164^{***}
	SEGEID	0.25067	0.0014	0.00007	-0.84643^{*}	6.5543

* at 5%, ** at 1% and *** at 10% significant

4.6 Comparison of Error Innovation Distributions for Fitness and Model Selection of Some GARCH Family Models on S&P500 Index Returns

The results of fitness and model selection according to log likelihood (LL) and Akaike information criteria (AIC) are shown in Table 7. We have examined several volatility models using SEGEID. According to the probability function and the lowest value of the Akaike information criteria (AIC), GJR-GARCH (1,1) was considered to be the best for different GARCH model families. The GJR-GARCH (1,1) model with SEGEID has surpassed other models according to the results of the overall model evaluation and the AIC minimum value as shown in Table 7 below.

Table 7: Comparison of the Error Innovation Distribution for Model Selection of Some GARCH Family Models on Standard and Poor-500 (S&P500) Index Returns

Models	Error Distributions	LL	AIC
GARCH (1,1)	SEGEID	147000.899	-13.0394
GJR-GARCH (1,1)	**SEGEID**	**454704.899**	**-17.0394**
EGARCH (1,1)	SEGEID	45009.8996	-9.0394
TGARCH (1,1)	SEGEID	46007.8861.143	-9.8284

bolded values are the highest value of likelihood function and the least value of AIC

4.7 Comparison of the GARCH Family Models estimated on SEGEID for Forecasting Performance on S&P500 Index Returns

Table 8 shows the root Mean square error (RMSE), which is the predictive performance of a model estimated using SEGEID. The lowest root Mean square error (RMSE) model was considered the best model to predict performance, and was determined by the different error innovation distributions. The results indicate that the SEG error innovation distribution with GJR-GARCH (1,1) has outperformed other volatility models in forecast performance, as taken into account in this research.

Table 8: Comparison of Error Innovation Distribution for Forecasting Evaluation of Some GARCH Family Models Based on S&P500 Index Returns

Model	Error Distributions	RMSE
GARCH (1,1)	SEGEID	0.00510
GJR-GARCH (1,1)	**SEGEID**	**0.00011**
EGARCH (1,1)	SEGEID	0.00360
TGARCH (1,1)	SEGEID	0.03660

RMSE- Root Mean Square Error, bolded values are the least RMSE

4.8 Discussion of findings

Analysis of S&P 500 indexes yields the lowest **AIC (-17.0394)** and root mean square error (0.00011), GJR-GARCH (1,1) and SEGEID, while GARCH (1,1), EGARCH (1,1) and TGARCH (1,1) yield the lowest AIC and root mean square error. In other words, the findings show that the GJR-GARCH (1,1) model outperforms the other three volatility models.

5. Conclusion

In this study, the GARCH family (1.1), GJR-GARCH family (1.1), E-GARCH family (1.1) and T-GARCH model families are used to create more adapted SEGEID error innovation distributions. SEGEID has improved over some of the existing error distributions having considered selection criterion of the volatility models and forecasting evaluation as seen in the result of the analysis and showed that, using the returns data of the S&P-500 index, the use of these error innovation distributions (GJR-GARCH (1,1) was more effective and predictive than the other volatility models studied in this study. This discovery is remarkable and a resource for both shareholders and the systemic community because the newly developed Error distribution can model all kinds of data irrespective of the situations (like war, Covid -19, election etc that may surround the data set) that may surround the data set

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