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# Information Contagion Within Social Networks in the Presence of Confirmatory Bias

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***Abstract:*** This paper examines the impact of confirmatory bias as it modifies the way in which information is disseminated within a small world network. A small world network is used as it has many of the characteristics found in social networks. Agents receive a private signal on the state of the world which is then adjusted following discussions with neighbours in a manner consistent with the social learning literature. It is found that the presence of confirmatory bias decreases the speed at which agents learn of changes in the state of the world and increases the level of memory in the system. When individuals suffer extreme levels of the bias, information cascades can emerge leading to long-term misalignment between the majority view of agents and the true state of the world.

Key words: Cognitive dissonance, confirmatory bias, information contagion, small world networks, social learning,
JEL classification: D82, D83

## 1. Introduction

People want to believe that they are smart, that they have made the right decision or that their beliefs reflect reality. When new information contradicts previously held beliefs, it triggers a tendency to feel psychologically uncomfortable. This phenomena, known as cognitive dissonance, was first proposed by Festinger (1957). Confirmatory bias, where new information is misinterpreted as supporting current beliefs, is one psychological bias which minimises this discomfort.

There is emerging evidence that conformity bias can impact economic behaviour in a predictable manner. In financial markets analysis, past forecast errors influence the current forecast (Friesen and Weller 2006). Prast and de Vor (2005) suggest that information filtering by investors may explain divergence between the euro and US dollar exchange rates despite a convergence in the growth rates of the two regions. Park *et al.* (2010) found that investors exhibit confirmatory bias when processing the information on the largest message board operator in South Korea. Outside of financial markets, Andrews *et al.* (2012) found that expert football pollsters significantly upgrade their beliefs about a team's quality when that team only slightly exceeds market expectations. It is also found that association with political parties, or voting for a political candidate, leads an individual to more favourably interpret the political actions of that party/ candidate (see for example, Kosnik 2005; Mullainathan and Washington 2009).

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This paper seeks to understand the impact of conformity bias on aggregate public opinion dynamics in a situation where agents share information within a social network to form their expectations on economic or social variables. When cognitive dissonance is incorporated into the model, agents experience the pain of regret when they receive information from neighbours that is inconsistent with their current opinion (or expectation). Individuals reduce cognitive dissonance by falling prey to confirmatory bias whereby there is a positive probability that new information, when inconsistent with the agent's current opinion, is misinterpreted as supporting that opinion. As long as individuals do not suffer extreme levels of confirmatory bias, it is found that its effect on the evolution of agents' aggregate public opinion is subtle. It induces a lag in the response time of the average agent's opinion to changes in the external environment as well as induces an increase in memory of the aggregate opinion of agents (i.e., the past now matters). It also appears to reduce short term volatility in the mean level of agent opinions. In cases where agents suffer extreme levels of conformity bias, information cascades arise resulting in long term misalignment of the aggregate opinion of agents from the true state of their external environment.

## 2. Literature Review

The existence of cognitive dissonance has been used to demonstrate market failure requiring government intervention (see for example the seminal paper by Akerlof and Dickens 1982) as well as well as giving rise to government failure (Brady *et al.* 1995). Cognitive dissonance has been incorporated into economics models, for example the labour market (Dickinson and Oxoby 2011; Goldsmith *et al.* 2004; Mullainathan and Washington 2009; Smith 2009), class (Oxoby 2003: 2004), and fairness and moral behaviour (Konow 2000; Matthey and Regner 2011; Rabin 1994).

One paper of particular relevance is that of Orlean (1995) which extends the social learning framework of Bikhchandani *et al.* (1992) to examine how interacting agents deal with environmental uncertainty.[1] Agents have an information set comprising of a private signal obtained in round one, an aggregate group opinion in the subsequent round, and their last opinion. Incorporating the last opinion in their information set introduces a form of cognitive dissonance so that their opinion provides an anchor when developing future beliefs. If agents believe that the majority of agents are imitators, then it is shown that agents act independently. When it is believed that the majority of agents are informed, then the dynamics of group opinion depends on the strength of belief in their own signal. If belief is strong, then a similar outcome to the independent case is obtained. If the collective belief in their own signal is weak, agents' views become polarised.

There are a number of approaches to modelling confirmatory bias within a theoretical framework. The first focuses on selective exposure. In this case, agents block out all new information which is inconsistent with current beliefs up to a threshold at which point, retaining initial beliefs becomes too costly to maintain and is consequently revised (see

[1] The model of Bikhchandani *et al.* (1992), as well as the majority of papers on information cascades that followed such as Orlean (1995), and this paper, are binary choice models. See De Vany and Lee (2001) for an extension of the binary choice model to a multiple choice framework.

Gilad *et al.* 1987). In a similar vein Gottlieb (2010) models confirmatory bias as selective awareness. A second approach posits that agents may misinterpret new information that conflicts with their current belief. For example, in Rabin and Schrag (1999), agents may misinterpret signals that conflict with their current belief with a probability $q$. Agents then process information in a Bayesian manner believing their misinterpreted signal to be unbiased. In Yariv (2002), confirmatory bias is modelled as agents selectively modifying new information to match previously held beliefs in order to maximise utility (utility is dependent, amongst other elements, on the difference between an agent's belief in the current and previous periods). Another approach is to model agents placing more (less) weight on information that confirms (is inconsistent) with current preferences (Ch'ng and Zaharim 2010; Pouget and Villeneuvez 2009).[2]

Recently, agent based models have been used to analyse the impact of confirmatory bias on financial market efficiency (Righi *et al.* 2012) and societal opinion dynamics (Ch'ng and Zaharim 2010). Righi *et al.* (2012) consider the impact of confirmatory bias in a single traded asset market in which agents share information about prices through social interaction but disregard new information that differs significantly from their own. The authors find that prices deviate from the asset's fundamental value in the long run thereby increasing informational inefficiency. Ch'ng and Zaharim (2010) model the spread of opinions of a product in a social network. Confirmatory bias is modelled as an agent placing more (less) weight on information that confirms (is inconsistent) with current preferences. Confirmatory bias also has a bias 'up/down' effect on agents' information preferences when relaying it to other agents depending on whether their utility is greater than, or less then, their expectation of the product. Ch'ng and Zaharim (2010) found that agents confirmatory bias leads to convergence of opinions. Finally, as agents bias up/down their preferences when sharing information, the spread of positive (negative) information is faster for a good (bad) product.

The remainder of this paper is structured as follows. Section 3 of this paper reviews the work of Rabin and Schrag (1999) in greater detail while Section 4 sets out the model and Section 5 the general results. Section 6 expands the basic model so that the strength of the confirmatory bias increases, the longer an individual holds the same view on the external environment. The final section summarises the paper and draws some conclusions.

## 3. Confirmatory Bias and its Effect on Bayesian Learning

Individuals strongly reacting to a condition of cognitive dissonance will typically have posterior distributions that are solely dependent on the information currently available to them. Rabin and Schrag (1999) explore the consequences of departing from Bayesian learning where agents having received new information inconsistent with current beliefs,

[2] When agents are subject to confirmatory bias, they tend to discard or modify new information that substantially differs from their priors. As noted by Righi *et al.* (2012), this compares to adaptive expectations where agents give more importance to the past (e.g. prices or inflation) than a rational agent. Aldashev *et al.* (2011) found that in financial markets, the presence of the two biases (adaptive expectations and confirmatory bias) can in certain circumstances, mitigate each other's effects.

are reluctant to shift away from these beliefs. In their model, there are two states of the world $x \in \{A,B\}$. An agent initially believes that each state is equally likely $P(x = A) = P(x = B) = 0.5$. In each round the agent receives a signal $s_t \in \{a,b\}$ that is correlated with the true state of the world according to $P(s_t = a|x = A) = P(s_t = b|x = B) = \theta > 0.5$.

It is assumed that agents may misinterpret signals that conflict with their current belief about the true SoW. Let $\sigma_t \in \{\alpha, \beta\}$ be the signal perceived by the agent. Therefore, if the agent's current belief is that $x = a$, and they receive a signal $s_t = b$ then they may, with probability $q$, misinterpret the signal as $a$ in which case $\sigma_t = \alpha$. In perceiving the signal to be $\alpha$, the agent believes to have received a signal $a$ and processes this information in a Bayesian manner as if the signal received was unbiased. When the signal confirms the initial belief, then there is no confirmatory bias. As noted by Rabin and Schrag(1999 : 49) "....the presence of confirmatory bias means that the agents' perceived signals are neither independent nor identically distributed."

Given the above assumptions, it is now possible to model the departure from Bayesian learning as a result of the conformation bias. Consider an agent that has received a set of signals $s^{t-1} = (s_1, \ldots\ldots, s_{t-1})$, with the corresponding sequence of perceived signals $\sigma^{t-1} = (\sigma_1, \ldots\ldots, \sigma_{t-1})$, and the belief based on $P(x = A|\sigma^{t-1}) > 0.5$. Let $\overset{*}{\theta}$ be the probability that the agent receives a signal confirming their belief in the SoW when in fact that belief is wrong. Let $\overset{**}{\theta}$ be the probability that the agent receives a signal confirming their belief in the SoW when that belief is true. Then:

$$\overset{*}{\theta} = P\left(\sigma_t = \alpha \middle| P\left(x = A \middle| \sigma^{t-1}\right) > 0.5, x = B\right) = P\left(\sigma_t = \beta \middle| P\left(x = B \middle| \sigma^{t-1}\right) > 0.5, x = A\right) \tag{1a}$$

$$\overset{**}{\theta} = P\left(\sigma_t = \alpha \middle| P\left(x = A \middle| \sigma^{t-1}\right) > 0.5, x = A\right) = P\left(\sigma_t = \beta \middle| P\left(x = B \middle| \sigma^{t-1}\right) > 0.5, x = B\right) \tag{1b}$$

As there is a probability $q$ that agents will misinterpret a signal that conflicts with current beliefs, there are now two paths in which the perceived signal corresponds to the SoW - where the signal confirms the belief and when the signal does not confirm the belief but is misread.

Therefore:

$$\overset{*}{\theta} = (1-\theta) + \theta \cdot q \tag{2a}$$

$$\overset{**}{\theta} = \theta + (1-\theta) \cdot q \tag{2b}$$

If $q = 0$, the agent is unbiased and follows Bayes law. In the other extreme, if $q = 1$, then the agent's first signal completely determines his belief for the remainder of the game. A number of propositions are established. It is found that for high $\theta$ and $q$ (close to one), agents are likely to be correct in their assessment of the SoW and they will likely be over confident in this belief. However, in the instance where the agent initially believes in the incorrect SoW and recently changed that belief to the correct SoW, they will be under confident. Where $\theta$ is close to 0.5 (i.e. the signal is very weak) or $q$ is close to one then there is high probability that the agent may incorrectly perceive the SoW even after an infinite number of signals are received.

## 4. The Model

There are $i, j \in I = \{1,\ldots\ldots, N\}$ agents which form a network of inter-connected agents whereby each agent is directly connected with only a subset of all other agents. Each direct connection is represented by a symmetric edge and information can only flow from one agent to another along these edges.[3] However, because there are no isolated agents in the network, information can flow from one agent to all other agents indirectly via third party connections. In this way the social network determines the information flow within the system. At the beginning of each round $t \in \{1,2,\ldots\ldots,T\}$, each agent receives a private binary signal $x \in X = \{0,1\}$ correlated with the SoW $v \in V = \{0,1\}$ according to $P(x_{i,t} = 0|v = 0) = P(x_{i,t} = 1|v = 1) = q > 0.5$. Each agent $i$ then undertakes a process to establish a view on the SoW.

This process involves considering the private signal they receive as well as the most recent view taken by each of the agents with which they have a connection. Agent *i's* signal is then adjusted in light of the interactions with connected agents and this becomes their view for round *t*. Let $n^i \subseteq I$ be the set of agents connected to *i* and $X_n^i$ be the set of opinions of those agents plus the private signal of *i*. The belief of each agent, being the prior probability of *V*, is now updated each round by forming the posterior of *V* given the knowledge gained through the agent's signal and conversation to connected agents according to:

$$P_{i,t}\left(V_t|X_{n,t}^i\right) = \frac{P\left(X_{n,t}^i|V_t\right) P(V_t)}{P\left(X_{n,t}^i\right)} \tag{3}$$

In this paper the social network is modelled as a modified ring lattice graph. Agents are located at nodes placed along the circumference of a circle and are initially connected to agents on either side.[4] The most simple ring lattice graph has two nearest neighbours, that is, each agent is connected to two others, these being the closet agent on either side. This ring lattice graph is then modified by increasing the level of randomness of the structure. Each edge is selected in turn and, with a probability *p*, the edge is removed and replaced with another symmetric edge with a randomly selected agent; The higher the value of *p*, the greater the degree of randomness in the network. When *p* is set to one, the network is defined as 'random'.

Consider the case where the edges coupled to agent *i* have not be modified so that $X_{n,t}^i = \left(x_{i-1,t}, x_{i,t}, x_{i+1,t-1}\right)$, where: $x_{i-1,t}$ represents the expectation formed by *i*-1 at time *t*, $x_{i,t}$ represents the private signal received by *i* at time *t* and $x_{i+1,t-1}$ represents the expectation formed by *i*+1 at time *t*-1. If agents *i*+1 and *i*-1 form an expectation that *v* = 0 and *i* receives a signal $x_{i,t} = 1$, then

$$P_i\left(v = 1|x_{i-1} = 0; x_i = 1; x_{i+1} = 0\right) = \frac{P\left(X_n^i|V\right) P(V)}{P\left(X_n^i\right)} \tag{4}$$

$$= \frac{P\left(x_i = 1|v = 1\right) P\left(v = 1|x_{i-1} = 0; x_{i+1} = 0\right)}{P\left(x_i = 1|v = 1\right) P\left(v = 1|x_{i-1} = 0; x_{i+1} = 0\right) + P\left(x_i = 1|v = 0\right) P\left(v = 0|x_{i-1} = 0; x_{i+1} = 0\right)}$$

[3] As the edge is symmetric, information can be transferred in either direction.

[4] See Watts (1999) for further information on small world networks.

If agent *i* processes this information in a rational manner, the agent will not follow their signal and form the view that $v = 0$.

Now consider what happens if agent *i* suffers from confirmatory bias. If an agent's private signal, or the public view of its neighbours, contradicts their current public view, there is a probability $\omega$ that it is misinterpreted. To simplify the analysis, the focus is only on the effect of confirmatory bias related to the processing of the agent's private signal and assume that the agent's previous public view is incorrect. As demonstrated by Rabin and Schrag (1999), there are two paths in which agent *i*'s private signal will correspond to the incorrect SoW. There is a probability (1- $q$) that they receive an incorrect signal or a probability $q$ that a correct signal is received and that they misread this signal. More formally, let $\bar{x}_i$ be the perceived signal. The relevant probability becomes[5]:

$$P(\bar{x}_{i,t}=1|\bar{x}_{i,t-1}=1;v=0)= \tag{5}$$
$$P(x_{i,t}=1|\bar{x}_{i,t-1}=1;v=0)+P(x_{i,t}=0|\bar{x}_{i,t-1}=1;v=0)\;\omega=(1-q)+q\cdot\omega>q$$

Further it can be shown that:

$$\begin{aligned}&P(\bar{x}_{i,t}=0|\bar{x}_{i,t-1}=0;v=0)=\\&\qquad P(x_{i,t}=0|\bar{x}_{i,t-1}=0;v=0)+P(x_{i,t}=1|\bar{x}_{i,t-1}=0;v=0)\;\omega=q+(1-q)\cdot\omega>q\end{aligned} \tag{6}$$

$$P(\bar{x}_{i,t}=0|\bar{x}_{i,t-1}=1;v=0)=P(x_{i,t}=0|\bar{x}_{i,t-1}=1;v=0)\;(1-\omega)=q\cdot(1-\omega)<q \tag{7}$$

Whether this impacts on the final decision of the agent depends on the view of its neighbours (and whether the views of neighbours are misinterpreted). In the absence of confirmatory bias, $\omega$ equals zero. Agent *i* will, therefore, always ignore their private signal and the problem reduces to that examined in Bowden and McDonald (2008). When $\omega$ equals one, the agent will always misinterpret both their own private signal as well as the public view of all agents to which agent *i* is connected. Consequently, the public view formed in the first round will determine their view for all subsequent rounds. More generally, if all agents suffer from confirmatory bias, then the probability of shifting away from current views falls, inducing a form of memory in the process of expectation formation as described by Equation 2.1. The stronger the bias, the higher is $\omega$ and the more likely agents will, in aggregate, retain the current public view.

There is a view in the literature that confirmatory bias is most prevalent when information received is ambiguous and open to interpretation. No attempt is made to capture this fact. However, according to Equation 5, it can be shown that overwhelming evidence will prevail. This produces a somewhat similar effect to linking $\omega$ to the degree of ambiguity of the information. It is also assumed that experts do not suffer from confirmatory bias and always follow their own private signal.[6]

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[5] Equation 5 holds only if $q<1/(2-\omega)$. As $q<0.5$ and $0\le\omega\le1$, this condition holds.

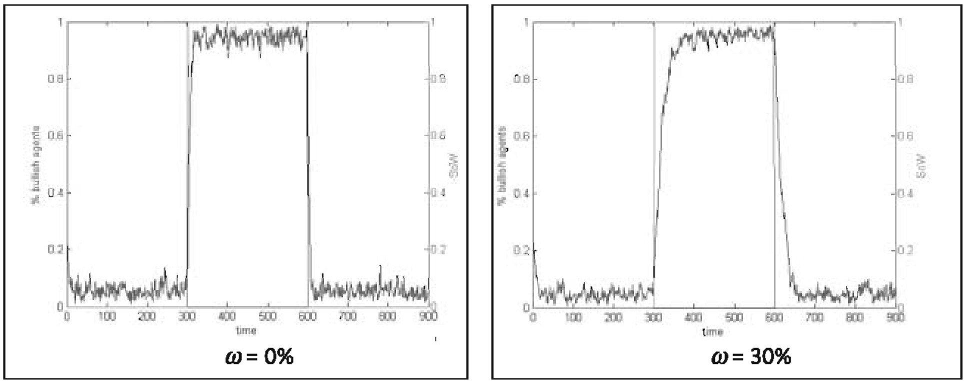
[6] If experts are also subject to confirmatory bias, then they would be slower to update their belief (as is the case for social learners). However, when social learners are impacted by confirmatory bias, this modifies the way in which information flows within the network which is the focus of this study. When experts fall prey to confirmatory bias, this will not alter the way in which information flows (as they do not engage in social learning) but rather slow the speed at which new information enters the network. As a result this paper does not analyse the case where experts suffer from confirmatory bias.

## 5. Results

Bowden and McDonald (2008) showed that when all agents engage in social learning, according to Equations 1 and 2, then learning ceases and an information cascade results. Initially, all agents learn the true SoW however, while any subsequent change may change the private signal received by the agent, it will be outweighed by the public view of connected agents. This is because agents do not share their private signal with others only their public view.[7] If one agent in the network (referred to as an expert agent) does not engage in social learning (i.e. they always follow their own signal) then the information cascade breaks and all agents will learn the SoW, as well as any subsequent changes to it. Adding expert agents increases the speed at which agents learn but at the expense of increased volatility. This is the case when agents process information using Bayesian updating.

When agents suffer from confirmatory bias, a path-dependant dimension is added as past beliefs matter. Figure 1 compares the evolution of agent expectations when $\omega$ is set at 0 and 30 per cent. For the purpose of the simulation when $t < 300$, $V$ is set exogenously to 0. As can be seen, agents herd around $x = 0$. At $t = 301$, $V$ is changed to 1. Agents switch their belief of $V$ to 1 (i.e. all but a few agents hold that $x = 1$ at any point in time). There is a minority of agents that believe that the $V = 0$ which arises when experts falsely believe that the SoW has changed and they then convince a minority of other agents through information contagion. At $t = 601$, $V$ is again changed and the same result occurs. When $\omega = 0$ per cent, then the speed at which agents learn of the change in the SoW is quick; however, when $\omega$ is increased to 30 per cent, the speed at which agents learn is significantly reduced.

In addition to the reduced speed of learning, the introduction of confirmatory bias appears to change the underlying process governing changes in the average view of agents. To establish whether this is the case, time series methods are used to determine




$\omega = 0\%$ $\omega = 30\%$

**Figure 1.** *Changing $\omega$ (5% of agents are experts)*

[7] This is consistent with the word-of-mouth literature where in communicating a view, hard evidence is not always provided. See for example Banerjee and Fudenberg (2004).

**Table 1.** A measure of path dependency

| | | $\omega = 0\%$ | $\omega = 10\%$ | $\omega = 30\%$ | $\omega = 50\%$ |
|---|---|---|---|---|---|
| Percentage of ARMA(1,1) models | | 31% | 63% | 84% | 91% |
| Percentage of models with ARCH/GARCH errors | | 19% | 3% | 0% | 6% |
| | $\alpha$ | 0.033 | 0.02 | 0.013 | 0.008 |
| | $\beta_1$ | 0.392 | 0.610 | 0.741 | 0.824 |
| | $\beta_2$ | NA | -0.196 | -0.252 | -0.321 |

*Note*: The parameters were calculated as the average of 32 simulations of the data using EViews. For $\omega$ = 0%,the remaining 69% of the simulations were estimated to be an AR(1) process. The parameter values of the sets of data not conforming with the preferred model (an AR(1) process for $\omega = 0$ and an ARMA(1,1) for the remaining levels of $\omega$) were excluded. Each of the simulations contained 1500 data points with the first 500 data points excluded from the econometric analysis. All parameters were significant to the 5% level. 5% of agents in the social network are experts.

if the error terms are dependent and, if so, the nature of this relationship. The results are presented in Table 1.

When $\omega = 0$, information flow within the network follows an AR(1) process where the average view of agents in *t* is dependent on the level in *t-1* plus an error term. Increasing the value of $\omega$ to 10 per cent changes this to an ARMA(1,1) process which has the standard form $y_t = \alpha + \beta_1 y_{t-1} + \varepsilon_t + \beta_2 \varepsilon_{t-1}$ (where $e_t$ represents white noise). It can be seen from Table 1 that as the value of confirmatory bias increases, the average view of agents becomes more dependent on past values. At the same time, the dependency on the MA(1) term $\beta_2$ is also increasing but the relationship is negative resulting in positive error terms in one period leading to a lower mean value of $y_t$ in the following period. It is therefore apparent that volatility is mean reverting.

As a final note, when $\omega = 0$ per cent, it is found that 20 per cent of the simulations contained ARCH or GARCH errors. However, when confirmatory bias is introduced only 3 simulations out of 96 conducted contained these errors. This may introduce complications in the analysis of time series data when it is generated by a decision making process influenced by confirmatory bias and the level of confirmatory bias is not constant over time. Consider data on financial prices. If buy and sell decisions are based on the view of the agent, depending on how the buying and selling impact on prices, these prices could experience periods where ARCH or GARCH errors are present, and other periods where they are not.

As a result of confirmatory bias, and the associated path dependency, agents are much slower to change their beliefs following the change in the SoW as it reduces the speed at which agents learn. It is found that the decrease in speed at which agents learn is dependent on the number of experts in the social network. In order to examine this relationship, the response time of agents to changes in the SoW is measured (using Monte Carlo methods) as the number of experts and $\omega$ is varied. The response time of agents is defined as the length of time from a change in the SoW to when the mean level of agent expectations reaches the average level of expectations if there had been no change in the SoW. The results are presented in Figure 2.

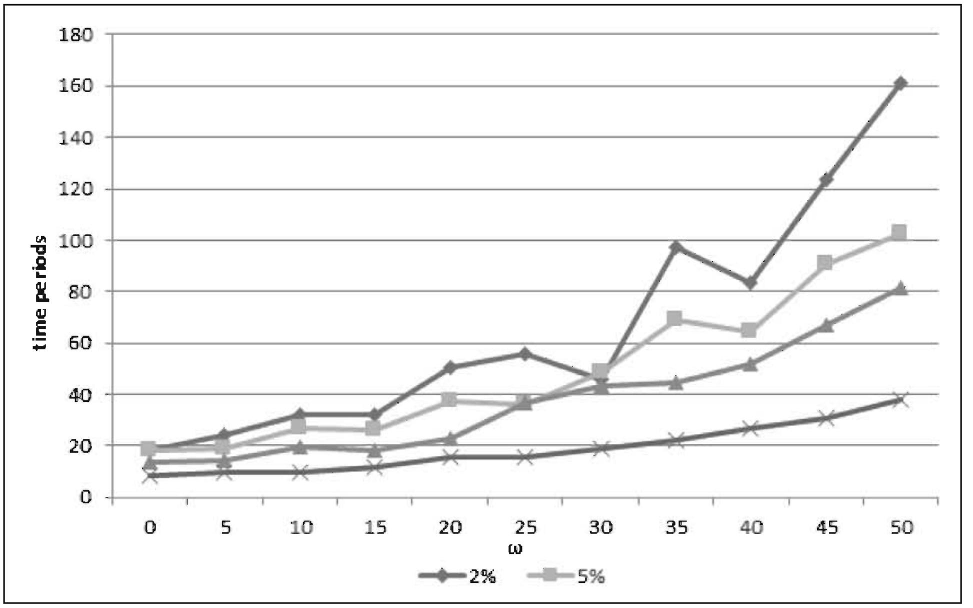



**Figure 2.** *Response time to changes in the State of the World (SoW)*

*Note*: Four cases are considered for different percentages of experts in the social network (from 2 to 20%). Each data point, represented as a cross, is the average of ten simulations of agents' response time to changing the SoW.

Two relationships emerge. The first is that the response time increases as $\omega$ increases. The second is that increasing the number of experts in the social network significantly decreases this effect. There are two reasons for this. As noted earlier, experts follow their own signal and do not engage in social learning. Therefore they are not affected by confirmatory bias. Second, when agents suffer from confirmatory bias, they in turn bias the information set of their neighbours. Increasing the number of experts reduces this effect.

The presence of confirmatory bias is often cited as an underlying cause of excess volatility. To examine if this is the case, the volatility in the mean level of agent expectations is measured as the level of $\omega$ and the number of expert agents (as a percentage of all agents) is increased.[8] Volatility is defined as the distribution ($\sigma = 1$) around the mean number of agents correctly guessing the SoW (holding the SoW constant for the period of the experiment).[9] The results are presented in Figure 3.

The volatility in expectations increases as the number of experts increase. However, volatility remains stable, and possibly decreases as $\omega$ increases. This is a rather surprising

[8] Consistent with the work of Bowden and McDonald (2008), volatility is measured as the distribution ($\sigma = 1$) around the mean number of agents correctly guessing the SoW.

[9] It is considered reasonable to define volatility in this way for the purpose of comparing different levels of $\omega$ as the distribution does not appear to be skewed. Further, the distribution does not suffer from the presence of ARCH or GARCH errors for positive levels of $\omega$.

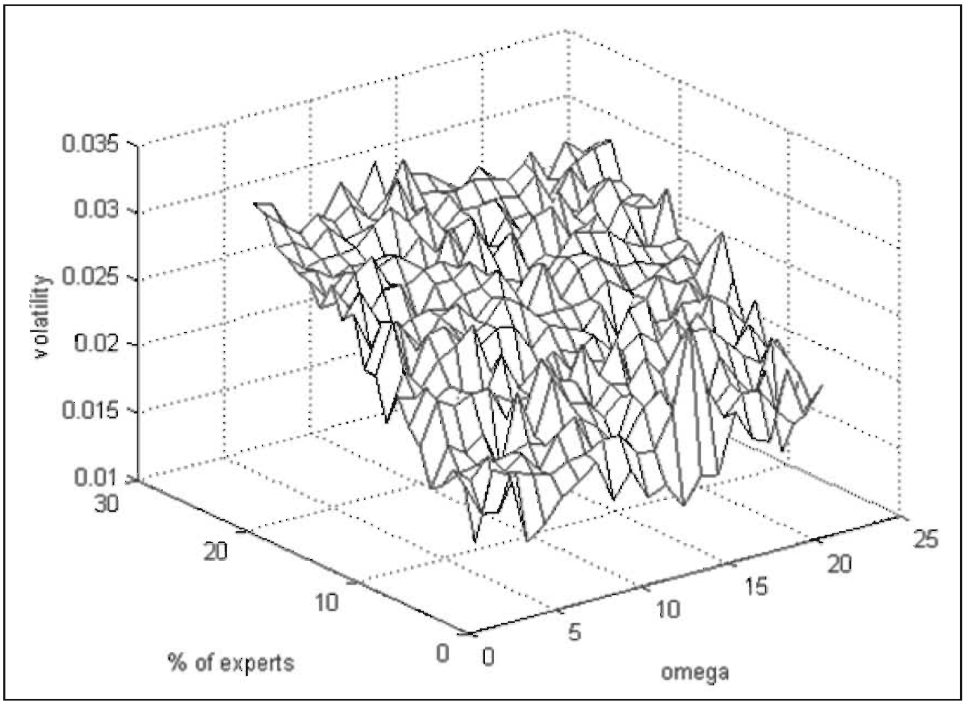



**Figure 3.** *Volatility in the mean level of agent expectations*

result and could well be a product of the mean reverting process associated with the negative $\beta_2$ value of the ARMA process as outlined in Table 1.

## 6. Strength of Attachment

Confirmatory bias can be thought of as the strength of attachment an agent has to a public view. In Section 5, the strength of confirmatory bias is exogenously set. This allows a more structured approach to analysing the effect of the bias on volatility, lag length and memory of expectations. However, in treating $\omega$ as an exogenous variable, it is not dependent on the strength of attachment the agent has to public view. The strength of attachment could depend on the strength of the agent's belief on which state is more likely such as surmised by Rabin and Schrag (1999).[10] Alternatively the strength of attachment could depend on the length of time that the agent has held a particular view. The first approach is most relevant when the external environment, represented as the SoW, does not change over time, as in Orlean (1995).

Let the agents information set containing historical and new data be represented by $h_{t-1}$ and $X_t$ respectively. Agents will calculate the probabilities associated with each

[10] Rabin and Schrag (1999) do not examine the implications of such an assumption, instead they set the strength of confirmatory bias as exogenous.

plausible SoW $V$ according to $P_t(V|X_t,h_{t-1})$ where $X_t$ or any element of $h_{t-1}$ may have been misinterpreted. If $X_{t+1}$ is inconsistent with $P_t(V|X_t,h_{t-1})$ > 0.5 for a given value of $V$, then there is a chance, based on the value of $P_t$, that it will be misinterpreted thereby re-enforcing the agents current belief and raising the value of $P_t$ rather than reducing it (which would have been the case had $x_{t-1}$ not been misinterpreted). If $t$ is large and the information set overwhelmingly indicates a given value of $V$, then the presence of confirmation bias is likely to have little effect on the outcome. However, if $t$ is small, then confirmatory bias is likely to distort the outcome away from that of the rational agent.[11] Because of the hysteresis properties associated with confirmatory bias, misinterpreting signals could have a lasting, and possibly permanent effect on the agent's beliefs including a complete misalignment of the beliefs with reality.

Now consider what would happen in a world where the environment changes. The value of the set of historical information $h_{t-1}$ would decrease relative to new information. Unless $h_{t-1}$ can provide some help in identifying structural changes in the environment, then $X_t$ becomes the best indicator of recent change. If the agents are concerned with identifying structural change, as they are in this paper, then $X_t$ becomes the only indicator of change. Given the strong weight placed on $X_t$ the existence of confirmatory bias can have a very strong effect on the outcome compared to the rational agents irrespective of the time period.

Consider now the alternative whereby the strength of attachment is a function of the time that the agent has held that particular public view. This is the approach adopted in this paper. The exact nature of this relationship could follow a number of functional forms. In this paper, it is modelled as according to the sum of a geometric progression:

$$\omega = f(\Omega,\alpha,\delta) = \begin{cases} \alpha \cdot \sum_{t=1}^{\Omega} \delta^{t-1}, & 1 \le \Omega < T \\ 0, & \Omega = 0 \end{cases} \tag{8}$$

For; $0 \le \alpha \le 1$; $0 \le \delta \le 1$.[12]

$\Omega$ represents the number of previous periods for which the agent's current public view on the SoW has remained unchanged. Once an agent changes the public view, $\Omega$ reverts to zero; if the view of the agent remains unchanged for the following round it increases to one. $\Omega$ continues to increase by one for each round that the view remains

[11] Consider the case where new information contradicts the current beliefs of the agent. If the set of information that forms the past history is large relative to the set of new information, and new and historic information are weighted equally, then the rational agent's beliefs will adjust to new information but the adjustment will be slight. If on the on the other hand, the set of information that forms the past history is of the same order of magnitude as new information then the adjustment will be more significant.

[12] In the absence of specific evidence of this relationship, the choice of function form is somewhat arbitrary (recognising that the sequence must converge to a limit as $\Omega$ increases). Possible alternatives to equation 8 could be $\alpha \cdot \log(\Omega)/\Omega^2$ or a hyperbolic function of the form *Tanh(Ω)*. Each of these functions will alter the speed of convergence; however, a similar result can easily be obtained by varying $\delta$ in equation 8.

unchanged. $\alpha$ is the initial strength of confirmatory bias when $t = 1$ and as a result must be between 0 and 1 (i.e., it represents the probability that a signal, inconsistent with current belief, is misread in the round $t = 1$). Finally, $\delta$ is the rate at which agents discount the strength of previous views.[13]

The assumption "that the strength of attachment is a function of time" has the disadvantage that it reduces the probability that agents form strong attachments to their view in the early periods.[14] However, in a model where past histories do not form part of the agent's set of information, this assumption should have less impact on the results. On the other hand, basing the strength of attachment on time rather than directly on the value of $P_t\left(V \middle| X_t, h_{t-1}\right)$ does have the advantage that agents can form a strong attachment to a view even if the evidence suggests the outcome is inconclusive (i.e., $P_t$ is approximately 50 per cent).[15]

Situations where the strength of confirmatory bias increases the longer that an agent holds a certain view or belief may arise in financial markets. For example, consider an investor who has experienced a long and deep correction. When the correction finally does end, the investor may become reluctant to believe it. Another example might be businesses operating at the end of a long period of growth in the economy. Management within the firm may view any downturn initially as a minor correction or even as a statistical error or anomaly. Only once when the economy moves well into recession does management begin to change their views. Having endogenised the level of confirmatory bias, the path dependency and speed at which agents learn is analysed and compared to the exogenous case. Table 2 outlines the statistical process when $\alpha = 30\%$, $\delta = 0.25$ and $\alpha = 30\%$, $\delta = 0.5$.

Comparing the results of Table 1 with Table 2, it appears that $\alpha = 30\%$, $\delta = 0.25$ produces a similar result to $\omega = 30\%$. Further, it is found that $\alpha = 30\%$, $\delta = 0.5$ and $\omega = 50\%$ produces similar preferred model specifications. Therefore, endogenising the level of confirmatory bias provides little additional insight into the changes in the underlying process than what has already been outlined in Section 5.

The speed at which agents learn of changes to the SoW is illustrated in Figure 4. Eight cases are compared. In the first four cases $\alpha$ is set at 10% while the number of experts increases from 2% to 20%. In the second four cases, $\alpha$ is set at 30%.

For $\alpha$ equal to 10 per cent, increasing $\delta$ has little effect on the length of time for agents to learn of changes in the external environment. However, when $\alpha$ is increased to

[13] To understand the mechanics associated with the discounting mechanism, consider an agent that has held the same public view for two periods. If they continue to hold that same view for an additional period (three rounds in total), then the increase in strength of conviction in their public view is greater than the situation when an agent has held the same view for three periods and continues to hold this view for a fourth consecutive period.

[14] That is, unless the agent in question has a strong susceptibility to confirmation bias.

[15] To the best of my knowledge, psychology literature has not directly addressed the issue of strength of association of individuals to a view, belief or decision as it relates to either the length of time they have held that view, or the probability they associate to their belief being correct. In practice, it may be difficult to determine if the strength of attachment is a result of the confidence an individual has in his/her belief, or the length of time the belief has been held, as in many circumstances they are likely to have the same effect.

**Table 2.** A measure of path dependency with ω endogenous and $\alpha$ = 30%

| | | $\delta$= 0.25 | $\delta$= 0.5 |
|---|---|---|---|
| Percentage of ARMA(1,1) models | | 84% | 94% |
| Percentage of models with ARCH/GARCH errors | | 3% | 3% |
| | $\alpha$ | 0.011 | 0.008 |
| | $\beta_1$ | 0.753 | 0.810 |
| | $\beta_2$ | -0.295 | -0.359 |

*Note*: The parameters were calculated as the average of 32 simulations of the data using EViews. The parameter values of the sets of data not confirming with the preferred model were excluded. Each of the simulations contained 1500 data points with the first 500 data points excluded from the econometric analysis. All parameters were significant to the 5% level. For $\delta$ = 0.75, 24 runs of 32 were found to be noise. 5% of agents were experts.

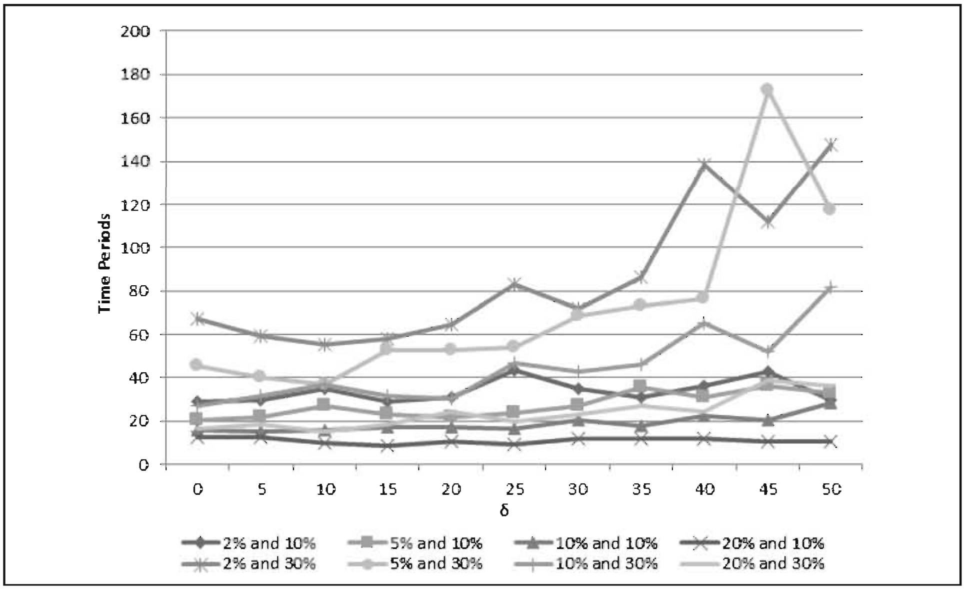



**Figure 4.** Response time to changes in the State of the World with $\omega$ endogenous.

30 per cent, levels of $\delta$ above 0.25 now impact on the speed of learning. A comparison of Tables 1 and 2, and Figures 2 and 4 show that when $\alpha$ is set to 30 per cent, changing $\delta$ has the same effect as changing $\omega$. However, for lower levels of $\alpha$, this is not the case and changing $\delta$ does not modify the way in which information spreads through the network. For a higher level of $\delta$ (such as $\alpha$ = 30 per cent and $\delta$ = 70 per cent), it is found that cascades emerge. When $\alpha$= 30 per cent and $\delta$= 70 per cent, then $\alpha$ (i.e. $\omega$) equals 1. At this level of confirmatory bias, agents always ignore any new signal that is inconsistent with current beliefs and agents enter into an information cascade. However, even for levels of

the bias marginally below the level of $\alpha = 30$ per cent, and $\delta = 70$ per cent, cascades failed to emerge.[16]

Cascades only emerge if the strength of the confirmatory bias is endogenously set and is very strong. This provides an additional avenue to the formation of information cascades to that found in Bowden and McDonald (2008) whereby certain network structures lead to the formation of information cascades.

## 6. Conclusion

This paper seeks to understand the impact of confirmatory bias as it modifies the way in which information is disseminated within a small world network. At the beginning of each round, all agents receive an imperfect signal on the state of their external environment. They then seek the views of their connected neighbours and adjust their signal following these discussions to form a public opinion. If agents receive a signal, or if an agent's neighbours have a public view that contradicts their own public view, the agent experiences cognitive dissonance. Individuals reduce cognitive dissonance by falling prey to confirmatory bias whereby there is a positive probability that new information, which is inconsistent with the agent's current public view, is misinterpreted as supporting the public view. Agents then update their view based on this possibly erroneous information using Bayes' Rule.

Confirmatory bias has two important and related effects - it decreases the speed at which agents learn of changes in their external environment and modifies the underlying process generated by the exchange of information between agents. In the absence of confirmatory bias, the mean level of expectations follow an AR(1) process. As the level of confirmatory bias is increased, the underlying process changes to an ARMA(1,1) with the AR(1) term becoming more positive but the MA(1) term becoming more negative. While the speed at which the system approaches equilibrium decreases, the system always settles into its equilibrium in finite time.

A second finding of the paper is that volatility remains stable (or, if anything, decreases slightly) as the level of confirmation bias increases. Finally, it is found that when confirmatory bias is exogenously set, then information always flows within the social network. However, when the confirmatory bias is endogenous (where the strength of the bias is dependent on the length of time the agent has held the same public view) then under certain conditions, information cascades can emerge.

## References


Akerlof, G.A. and W.T. Dickens. 1982. The economic consequences of cognitive dissonance. *American Economic Review* **72(3)**: 307-319.

Aldashev, G., T. Carletti and S. Righi. 2011. Follies subdued: Informational efficiency under adaptative expectations and confirmatory bias. *Journal of Economic Behaviour* and Organisation **80(1)**:110-121.


[16] 100 runs of the model for 900 rounds (similar to Figure 1) were conducted for the parameter values $\alpha = 30\%/\delta = 65\%$ and $\alpha = 30\%/\delta = 70\%$. None of the runs for $\delta = 65\%$ resulted in an information cascade while all of the runs for $\delta = 70\%$ did.

Andrews, R.J., T.D. Loga and M.J. Sinkey. 2012. Identifying Confirmatory Bias in the Field: Evidence from a Poll of Experts, NBER Working Paper No. 18064.

Bannerjee, A. and D. Fudenberg. 2004. Word-of-mouth learning. *Games and Economic Behaviour* **46(1)**:1-22.

Bikhchandani, S., D. Hirshleifer and I. Welch. 1992. A theory of fads, fashion, custom, and cultural change in informational cascades. *Journal of Political Economy* **100(5)**: 992-1026.

Bowden, M. and S. McDonald. 2008. The impact of interaction and social learning on aggregate expectations. *Computational Economics* **31(3)**: 289-306.

Brady, G.L., J.R. Clark and W.L. Davis. 1995. The political economy of dissonance. *Public Choice* **82(1)**: 37-51.

Ch'ng, K.-S.and N.M. Zaharim. 2010. Confirmation bias and convergence of beliefs: An agent-based model approach. *Malaysian Journal of Economic Studies* **47(1)**: 19-31.

De Vany, A. and C. Lee. 2001. Quality signals in information cascades and the dynamics of the distribution of motion picture box office revenues. *Journal of Economic Dynamics and Control* **25(3)**: 593-614.

Dickinson, D.L. and R.J. Oxoby. 2011. Cognitive dissonance, pessimism, and behavioral spillover effects. *Journal of Economic Psychology* **32(3)**: 295-306.

Festinger, F. 1957. *A Theory of Cognitive Dissonance*. Stanford, CA: Stanford University Press.

Friesen, G. and P.A. Weller. 2006. Quantifying cognitive biases in analyst earnings forecasts. *Journal of Financial Markets* **9(4)**: 333-365.

Gilad, B., S. Kaish and P.D. Loeb. 1987. Cognitive dissonance and utility maximization: A general framework. *Journal of Economic Behavior and Organization* **8(1)**: 61-73.

Goldsmith, A.H., S. Sedo, W. Darity and D. Hamitlon. 2004. The labour supply consequences of perceptions of employer discrimination during search and on-the-job: Integrating neoclassical theory and cognitive dissonance. *Journal of Economic Psychology* **25(1)**: 15-39.

Gottlieb, R. 2010. Will you never learn? Self-deception and biases in information processing. Princeton University.

Konow, J. 2000. Fair shares: Accountability and cognitive dissonance in allocation decisions. *American Economic Review* **90(4)**: 1072-1091.

Kosnik, L. 2005. Refusing to budge: A confirmatory bias in decision making. Department of Economics, University of Missouri.

Matthey, A. and T. Regner. 2011. Do I really want to know? A cognitive dissonance-based explanation of other-regarding behavior. *Games* **2(1)**: 114-135.

Mullainathan, S. and E. Washington. 2009. Sticking with your vote: Cognitive dissonance and voting. *American Economic Journal: Applied Economics* **1(1)**.

Orlean, A. 1995. Bayesian interactions and collective dynamics of opinion: Herd behavior and mimetic contagion. *Journal of Economic Behavior and Organization* **28(2)**: 257-274.

Oxoby, R.J. 2003. Attitudes and allocations: Status, cognitive dissonance, and the manipulation of attitudes. *Journal of Economic Behavior and Organization* **52(3)**: 365-385.

Oxoby, R.J. 2004. Cognitive dissonance, status and growth of the underclass. *Economic Journal* **114(498)**: 727-749.

Park, J., P. Konana, B. Gu, A. Kumar and R. Raghunathan. 2010. Confirmation bias, overconfidence, and investment performance: Evidence from stock message boards, McCombs Research Paper Series.

Pouget, S. and S.Villeneuvez. 2009. Price formation with confirmation bias. Toulouse School of Economics, University of Toulouse.

Prast, H.M.and M.P.H. de Vor. 2005. Investor reactions to news: A cognitive dissonance analysis of the euro-dollar exchange rate. *European Journal of Political Economy* **21(1)**: 115-141.

Rabin, M. 1994. Cognitive dissonance and social change. *Journal of Economic Behavior and Organization* **23(2)**: 177-194.

Rabin, M. and J.L.Schrag. 1999. First impressions matter: A model of confirmatory bias. *Quarterly Journal of Economics* **114(1)**: 37-82.

Righi, S., T. Carletti and G. Aldashev. 2012. Behavioral biases and informational inefficiency in an agent-based financial market. *Proceedings of European Social Simulation Association Conference*, University of Salzburg, Austria.

Smith, J. 2009. Cognitive dissonance and the overtaking anomaly: Psychology in the principal–agent relationship. *Journal of Socio-Economics* **38(4)**: 684-690.

Watts, D.J. 1999. *Small Worlds : The Dynamics of Networks between Order and Randomness*. New Jersey: Princeton University Press.

Yariv, L. 2002. I'll see it when i believe it - a simple model of cognitive consistency. Cowles Foundation Yale University Cowles Foundation Discussion Papers 1352.