Stability Analysis of SCIR-SI Compartmental Model for Meningococcal Meningitis Disease between Two Regions

Rifaldy Fajar[1] and Eminugroho Ratna Sari[1]*

1Department of Mathematics Education, Universitas Negeri Yogyakarta, Yogyakarta, Indonesia.
*Corresponding author: eminugroho@uny.ac.id, rifaldy.fajar@student.uny.ac.id
Received: 30th Jan 2019 Accepted: 10th Jul 2019 Published: 28th Aug 2019
DOI: https://doi.org/10.22452/mjs.vol38no2.6

ABSTRACT Mathematical research about Meningococcal meningitis disease usually focused in one region, however *the spread of this disease also can be caused by residents who visiting epidemic region. So,* the purposes of this study are (1) to establish a mathematical model of Meningococcal meningitis disease between two regions, particularly Indonesia and Saudi Arabia, (2) to analyze the stability of each equilibrium points, (3) and to explain the model simulation of Meningococcal meningitis with the effect of vaccination on populations from Indonesia. The disease-free equilibrium point is locally asymptotically stable when the reproduction number is less than one and unstable when the reproduction number is more than one. In addition to the stability of the locally asymptotically stable endemic equilibrium point when the reproduction number is more than one. Based on the model simulation, if the level of vaccination is higher, then the infected class on populations from Indonesia and Inffected class from Saudi Arabia will decrease tend to zero. The vaccination program can be used to control the transmission of Meningococcal meningitis disease. It also can be useful to monitor the efficiency of vaccination program in the country.

Keywords: *Meningococcal meningitis,* compartmental model, Indonesia, Saudi Arabia

INTRODUCTION

Meningococcal meningitis, caused by *Neisseria meningitides,* is a serious form of meningitis and has the highest mortality rate if untreated. In addition, meningitis caused by this type of bacteria is reported to have a mortality rate in developed countries with a percentage of about 70−80% before treatment (Martcheva & Crispino-O'Connell, 2003). The ratio between epidemic and mean endemic incidence rates is generally higher in developing countries than in developed (World Health Organization, 2002). It happens relatively rarely, the incidence is less than 5 cases over 100,000 populations (Centers for Disease Control and Prevention, 2015).

Meningococci are transmitted by droplets from the respiratory tract, and infected individuals can transmit the bacteria up to 24 hours after the initiation of antibiotic treatment. Although colonization occurs in more than 10% of adults, rates as high as 42% have been observed among adolescents and young adults due to the social behavior of these populations i.e. kissing, smoking, and alcohol consumption that predispose them to transmission (Martinez *et al.,2013).*

N. meningitidis lives exclusively in the human upper respiratory tract and it is transmitted between hosts via oral secretions or direct contact. By means of their pili, meningococci adhere selectively to non-ciliated columnar cells of the nasopharynx, starting to multiply and colonize the site of mucosal attachment. They can survive even if phagocytosed by epithelial cells, from which they can escape to reach the submucosal or directly

invade damaged epithelial surfaces. It has been reported that diplococci colonies can be carried for several months even more than one year (Hung & Christodoulides, 2013).

Meningococcal meningitis is an infectious disease that is often carried by Hajj or Umrah where the disease occurs acutely and rapidly spreads to the congregation (Yezli, Assiri, Alhakeem, Turkistani, & Alotaibi, 2016). Based on the data from the World Health Organization, countries in the African continent are still an endemic country of meningitis disease. Given the geographical position of Saudi Arabia whose position lies precisely close to the African Continent, it is assumed that this area becomes an area with a high endemic prevalence such that the potential for contagious diseases for hajj jama'ah or umrah can occur quickly. In the other side, the number of Hajj or Umrah originating from Indonesia tends to increase every year and hence the potential spread of this disease can occur in Indonesia. Based on data from the Ministry of Religious Affairs of the Republic of Indonesia (2015), the average number of umrah from Indonesia is 195 people per day and the average number of hajis from Indonesia is 154,000 people per year.

Vaccination strategies used to manage meningococcal disease vary based on specific conditions within a region or country and the needs of the target population. For instance, routine, age-based prophylactic immunization is often used when typical, or endemic, disease rates prevail in specific age groups. In contrast, mass vaccination is often deployed under epidemic conditions, defined as an increase in cases compared with baseline endemic conditions (Vuocolo, et al., 2018).

Meningitis vaccine is a mandatory vaccine that must be done by prospective pilgrims to protect the risk of contracting meningococcal meningitis. Meningitis vaccine contains antigens, a substance that stimulates the immune system to form antibodies and fight the bacteria that cause meningitis, hence vaccine can reduce the risk of meningococcal Meningitis disease. Mathematics research about meningitis vaccine using control had done by Asamoah, *et al.* (2018). Blyuss (2016) also had discussed temporary population immunity to meningitis disease which can be useful to measure the efficiency of vaccines.

Furthermore, usually, mathematical research about meningitis is discussed in one region. For example Irving, et al. (2012) who had discussed meningitis in the Africa, and also Wiah (2010) who had discussed the disease in the Ghana. Therefore, it is logic if the discussion of meningitis is divided into two regions, for example between Indonesia and Saudi Arabia.

In this paper, a new model will be developed between the two regions in which case Indonesia (INA) and Saudi Arabia (KSA) use the population compartment $SCIR - SI$. This is done to see how the differences in population dynamics in each region caused by population movement. The displacement of the population is Indonesian citizens who will travel to Saudi Arabia as an umrah or hajj pilgrim, and the citizens of Saudi Arabia who also perform the pilgrimage.

MODEL FORMULATION AND ANALYSIS

There are six groups of individuals to be used, namely the number of susceptible populations present in Indonesia (S_{INA}), the number of bacterial carriers present in Indonesia (C_{INA}), the number of infected populations present in Indonesia (I_{INA}), the number of recovering populations Indonesia (R_{INA}), the number of susceptible populations present in Saudi Arabia (S_{KSA}), and the number of infected populations present in Saudi Arabia (I_{KSA}).

To simplify the process of modeling the spread of *Meningococcal meningitis* between Indonesia (INA) and Saudi Arabia (KSA) regions, assumptions are made. The assumptions used in this study are:

1. There are susceptible and infected individuals in both areas, especially in Indonesia, there are individual carriers of bacteria (carrier).
2. There are susceptible individuals who are given vaccinations in Indonesia and vaccinated individuals are not affected by the disease and fall into the cured category.
3. There are susceptible individuals, carriers of bacteria, infected and cured who died naturally in Indonesia.
4. There are susceptible and infected individuals who die naturally in Saudi Arabia.
5. The individual rate of all classes who die naturally for both regions is the same.
6. Individuals infected can recover in Indonesia.
7. Contacts between the two countries occurred during the Hajj and Umrah season.

The incubation period of meningococcal disease is 3 to 4 days, with a range of 2 to 10 days. Meningitis is the most common presentation of invasive meningococcal infection (meningococcal disease) and results from hematogenous dissemination of the organism (Asamoah, et al. 2018). So, time unit (*t*) is assumed to be calculated in weeks because the incubation period may exceed seven days (one week). The variables and parameters used in this modeling are:

Table 1. Variables and Parameters.

Variable	Definition	Range
S_{INA}	The number of susceptible populations from Indonesia	$S_{INA} \geq 0$
C_{INA}	The number of bacterial carrier population from Indonesia	$C_{INA} \geq 0$
I_{INA}	The number of infected populations from Indonesia	$I_{INA} \geq 0$
R_{INA}	The number of recovered population from Indonesia	$R_{INA} \geq 0$
S_{KSA}	The number of susceptible populations from Saudi Arabia	$S_{KSA} \geq 0$
I_{KSA}	The number of infected populations from Saudi Arabia	$I_{KSA} \geq 0$
Parameter		
A	The number of prospective Hajj / Umrah pilgrims from Indonesia	$A > 0$

β	The number of prospective Hajj / Umrah pilgrims from Saudi Arabia	$\beta > 0$
μ	The rate of individuals experiencing natural deaths in all groups	$\mu > 0$
π	The average number of contacts between the S_{INA} population and the I_{KSA} population	$\pi > 0$
τ	The average number of contacts between the S_{KSA} population and the I_{INA} population	$\tau > 0$
β_1	Probability of disease transmission from I_{KSA} population to S_{INA} population	$0 \le \beta_1 \le 1$
β_2	Probability of disease transmission from I_{INA} population to S_{KSA} population	$0 \le \beta_2 \le 1$
φ	Effectiveness of treatment on infected individuals	$\varphi > 0$
k	Recovery rate of infected individuals	$k > 0$
v	The transfer rate in the individual carriers of the bacteria (carrier) becomes an infected individual	$v > 0$
θ	Proportion of vaccinated susceptible individuals in Indonesia	$0 \le \theta \le 1$

According to the data in Table 1 and based on assumptions, the compartment diagram of the spread model of the *Meningococcal meningitis* disease can be depicted in Figure 1 as follows:

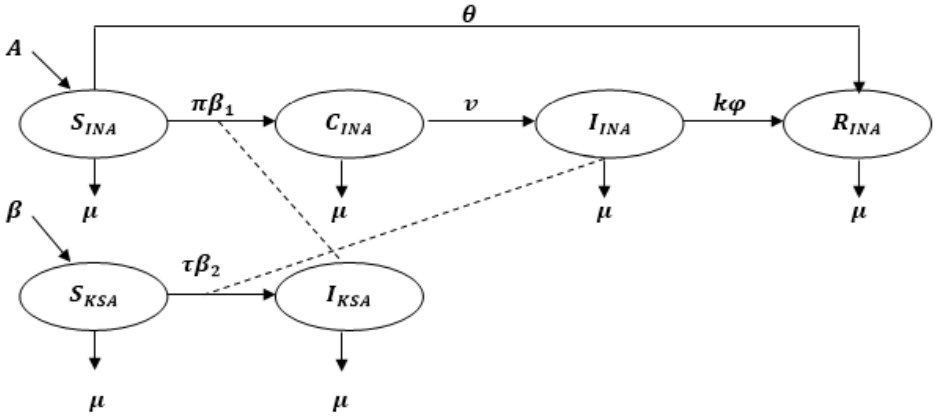

Figure 1. Compartment diagram of the spread model of *Meningococcal meningitis* disease between INA and KSA regions.

According to Figure 1, obtained mathematical model is as follows:

$$\frac{dS_{INA}(t)}{dt} = A(1-\theta) - \pi\beta_1 S_{INA} I_{KSA} - (\theta + \mu) S_{INA}$$

$$\frac{dC_{INA}(t)}{dt} = \pi\beta_1 S_{INA} I_{KSA} - (\mu + v) C_{INA}$$

$$\begin{aligned}
\frac{dI_{INA}(t)}{dt} &= vC_{INA} - (\mu + k\varphi) I_{INA} \\
\frac{dR_{INA}(t)}{dt} &= k\varphi I_{INA} + \theta S_{INA} - \mu R_{INA} \\
\frac{dS_{KSA}(t)}{dt} &= \beta - (\tau\beta_2 I_{INA} + \mu) S_{KSA} \\
\frac{dI_{KSA}(t)}{dt} &= \tau\beta_2 S_{KSA} I_{INA} - \mu I_{KSA}
\end{aligned} \tag{1}$$

with the domains of the variables in the model are

$$\Omega = \{(S_{INA}, C_{INA}, I_{INA}, R_{INA}, S_{KSA}, I_{KSA}) \in \mathrm{R}^6 : S_{INA}, C_{INA}, I_{INA}, R_{INA}, S_{KSA}, I_{KSA} \geq 0\}$$

and parameters employed in the model are $A, \beta, \pi, \tau, \mu, v, k, \varphi$ all of them are positive and $0 \leq \beta_1 \leq 1, 0 \leq \beta_2 \leq 1$, and $0 \leq \theta \leq 1$.

Model Transformation

Model transformation is used to analysis the solution behavior that can be easier. In this article, system (1) needs to be simplified by scaling, namely by changing the system (1) to form a proportion between the number of individuals in a subpopulation with the total population. Lemma 1 shows us how to get the proportion of INA populations.

Lemma 1

Given the initial value $F(0) \geq 0$, with $F(t) = (S_{INA}, C_{INA}, I_{INA}, R_{INA})$. The solution of the model (1) for the INA population is non−negative for all $t > 0$. Furthermore $\lim_{t \to \infty} sup N_{INA}(t) \leq \frac{A(1-\theta)}{\mu}$ with $N_{INA}(t) = S_{INA}(t) + C_{INA}(t) + I_{INA}(t) + R_{INA}(t)$.

Proof

Based on system (1),

$$\begin{aligned}
\frac{dN_{INA}}{dt} &= \frac{dS_{INA}}{dt} + \frac{dC_{INA}}{dt} + \frac{dI_{INA}}{dt} + \frac{dR_{INA}}{dt} \\
&= A(1-\theta) - \pi\beta_1 S_{INA} I_{KSA} - (\theta + \mu) S_{INA} + \pi\beta_1 S_{INA} I_{KSA} - \\
&\quad (\mu + k\varphi) I_{INA} + k\varphi I_{INA} + \theta S_{INA} - \mu R_{INA} \\
&= A(1-\theta) - \mu S_{INA} - \mu C_{INA} - \mu I_{INA} - \mu R_{INA} \\
&= A(1-\theta) - \mu (S_{INA} + C_{INA} + I_{INA} + R_{INA})
\end{aligned}$$

Because of $N_{INA}(t) = S_{INA}(t) + C_{INA}(t) + I_{INA}(t) + R_{INA}(t)$ we obtained

$$\frac{dN_{INA}}{dt} = A(1-\theta) - \mu N_{INA} \ . \tag{2}$$

The value of $\frac{dN_{INA}}{dt} < 0$ if $N_{INA} \geq \frac{A(1-\theta)}{\mu}$. On the other hand, $\frac{dN_{INA}}{dt} > 0$ if $N_{INA} \leq \frac{A(1-\theta)}{\mu}$.

So the least upper bound N_{INA} is $\frac{A(1-\theta)}{\mu}$. Model in system (1) describes the human population (INA) infected with *meningococcal meningitis*, it means that if all variables are non-negative for $t \geq 0$, then the solution with positive initial value will always remain positive $(t > 0)$.

Based on Equation (2), it is called the rate of change of total population in Indonesia, as $t \to \infty$ the solution N_{INA} approaches $\frac{A(1-\theta)}{\mu}$. Furthermore, Lemma 2 shows us how to get the proportion of KSA populations.

Lemma 2

Given the initial value $F(0) \geq 0$, with $F(t) = (S_{KSA}, I_{KSA})$. The solution of the model (1) for the KSA population is non−negative for all $t > 0$. Furthermore $\lim_{t\to\infty} sup N_{KSA} \leq \frac{\beta}{\mu}$ with $N_{KSA}(t) = S_{KSA}(t) + I_{KSA}(t)$.

Proof

Based on system (1),

$$\frac{dN_{KSA}}{dt} = \frac{dS_{KSA}}{dt} + \frac{dI_{KSA}}{dt}$$

$$= \beta - (\tau\beta_2 I_{INA} + \mu) S_{KSA} + \tau\beta_2 S_{KSA} I_{INA} - \mu I_{KSA}$$

$$= \beta - \tau\beta_2 I_{INA} S_{KSA} - \mu S_{KSA} + \tau\beta_2 S_{KSA} I_{INA} - \mu I_{KSA}$$

$$= \beta - \mu S_{KSA} - \mu I_{KSA}$$

$$= \beta - \mu(S_{KSA} + I_{KSA})$$

Because of $N_{KSA}(t) = S_{KSA}(t) + I_{KSA}(t)$ we obtained

$$\frac{dN_{KSA}}{dt} = \beta - \mu N_{KSA} \tag{3}$$

The value of $\frac{dN_{KSA}}{dt} < 0$ if $N_{KSA} \geq \frac{\beta}{\mu}$. On the other hand, $\frac{dN_{KSA}}{dt} > 0$ if $N_{KSA} \leq \frac{\beta}{\mu}$. So, the least upper bound N_{KSA} is $\frac{\beta}{\mu}$. Model in system (1) describes the human population (KSA) infected with *meningococcal meningitis*, it means that if all variables are non-negative for $t \geq 0$, then the solution with positive initial value will always remain positive $(t > 0)$.

Based on Equation (3), it is called the rate of change of total KSA population. As $t \to \infty$, the solution N_{KSA} approaches $\frac{\beta}{\mu}$. It means from Lemma 1 and 2, we can define the subset T by the equations $S_{INA} + C_{INA} + I_{INA} + R_{INA} = \frac{A(1-\theta)}{\mu}$ and $S_{KSA} + I_{KSA} = \frac{\beta}{\mu}$. It is an invariant region for system (1), since $N_{INA} \to \frac{A(1-\theta)}{\mu}$ and $N_{KSA} \to \frac{\beta}{\mu}$, all paths approach *T*. Therefore, it is enough to analyse the asymptotic behaviour of solution of system (1) in this invariant set.

The INA populations and KSA populations remain constant in *T*, then, without loss of generality, we can use the proportions

$$s_{INA} = \frac{S_{INA}}{\frac{A(1-\theta)}{\mu}}, c_{INA} = \frac{C_{INA}}{\frac{A(1-\theta)}{\mu}}, i_{INA} = \frac{I_{INA}}{\frac{A(1-\theta)}{\mu}}, r_{INA} = \frac{R_{INA}}{\frac{A(1-\theta)}{\mu}},$$

and

$$s_{KSA} = \frac{S_{KSA}}{\frac{\beta}{\mu}}, i_{KSA} = \frac{I_{KSA}}{\frac{\beta}{\mu}}.$$

Using these proportions, system (1) in the invariant space *T* can be written as follows

$$\begin{aligned}
\frac{ds_{INA}}{dt} &= \mu - \pi\beta_1 s_{INA} i_{KSA}\left(\frac{\beta}{\mu}\right) - (\theta + \mu) s_{INA} \\
\frac{dc_{INA}}{dt} &= \pi\beta_1 s_{INA} i_{KSA}\left(\frac{\beta}{\mu}\right) - (\mu + \nu) c_{INA} \\
\frac{di_{INA}}{dt} &= \nu c_{INA} - (\mu + k\varphi) i_{INA} \\
\frac{dr_{INA}}{dt} &= k\varphi i_{INA} + \theta s_{INA} - \mu r_{INA} \\
\frac{ds_{KSA}}{dt} &= \mu - \tau\beta_2 i_{INA}\left(\frac{A(1-\theta)}{\mu}\right) s_{KSA} - \mu s_{KSA} \\
\frac{di_{KSA}}{dt} &= \tau\beta_2 s_{KSA} i_{INA}\left(\frac{A(1-\theta)}{\mu}\right) - \mu i_{KSA}
\end{aligned} \tag{4}$$

System (4) which is a nonlinear equation system is the transformation result of the mathematical modeling of the spread of *Meningococcal meningitis* disease in the system (1). Now, we give a more analysis of the system (4).

Equilibrium Points

Equilibrium points of system (2) are obtained if (Perko, 2001)

$$\frac{ds_{INA}}{dt}=\frac{dc_{INA}}{dt}=\frac{di_{INA}}{dt}=\frac{dr_{INA}}{dt}=\frac{ds_{KSA}}{dt}=\frac{di_{KSA}}{dt}=0$$

Theorem 1

i. If $i_{INA}=c_{INA}=i_{KSA}=0$, then system (4) has disease free equilbrium point

$$E_0=\left(s_{INA}^{*},0,0,r_{INA}^{*},s_{KSA}^{*},0\right)$$

where $s_{INA}^{*}=\frac{\mu}{\theta+\mu}, r_{INA}^{*}=\frac{\theta}{\theta+\mu}$ and $s_{KSA}^{*}=1.$

ii. If $i_{INA}\neq c_{INA}\neq i_{KSA}\neq 0$, then system (4) has the endemic equilibrium point

$$E_1=\left(s_{INA}^{**},c_{INA}^{**},i_{INA}^{**},r_{INA}^{**},s_{KSA}^{**},i_{KSA}^{**}\right)$$

where $s_{INA}^{**}=\frac{\mu\left(R_0+M\right)}{\left(\theta+\mu\right)R_0\left(M+1\right)}$, $c_{INA}^{**}=\frac{\mu B_1\left(R_0-1\right)}{B_2R_0\left(M+1\right)}$, $i_{INA}^{**}=\frac{v\mu B_1\left(R_0-1\right)}{B_3R_0\left(M+1\right)}$,

$r_{INA}^{**}=\frac{k\varphi vB_1\left(R_0-1\right)}{B_3R_0\left(M+1\right)}+\frac{\theta\left(R_0+M\right)}{\left(\theta+\mu\right)R_0\left(M+1\right)}$, $s_{KSA}^{**}=\frac{M+1}{R_0+M}$, $i_{KSA}^{**}=\frac{R_0-1}{R_0+M}$.

Proof

From each equation in system (4) with the right-hand side equal to zero, then system (4) can be written as follow

$$\frac{ds_{INA}(t)}{dt}=\mu-\pi\beta_1s_{INA}i_{KSA}\left(\frac{\beta}{\mu}\right)-\left(\theta+\mu\right)s_{INA}=0 \tag{5.1}$$

$$\frac{dc_{INA}(t)}{dt}=\pi\beta_1s_{INA}i_{KSA}\left(\frac{\beta}{\mu}\right)-\left(\mu+v\right)c_{INA}=0 \tag{5.2}$$

$$\frac{di_{INA}(t)}{dt}=vc_{INA}-\left(\mu+k\varphi\right)i_{INA}=0 \tag{5.3}$$

$$\frac{dr_{INA}(t)}{dt}=k\varphi i_{INA}+\theta s_{INA}-\mu r_{INA}=0 \tag{5.4}$$

$$\frac{ds_{KSA}(t)}{dt}=\mu-\tau\beta_2i_{INA}\left(\frac{A(1-\theta)}{\mu}\right)s_{KSA}-\mu s_{KSA}=0 \tag{5.5}$$

$$\frac{di_{KSA}(t)}{dt}=\tau\beta_2s_{KSA}i_{INA}\left(\frac{A(1-\theta)}{\mu}\right)-\mu i_{KSA}=0 \tag{5.6}$$

i. Since $i_{INA}=c_{INA}=i_{KSA}=0$, from equation (5.1) is obtained

$$s_{INA}=\frac{\mu}{\left(\theta+\mu\right)} \tag{6}$$

Since $i_{INA} = c_{INA} = i_{KSA} = 0$, then substitution (6) into equation (5.4) then it is obtained

$$r_{INA} = \frac{\theta}{(\theta + \mu)} \tag{7}$$

Since $i_{INA} = c_{INA} = i_{KSA} = 0$, then the equation (5.5) obtained

$$s_{KSA} = 1 . \tag{8}$$

Based on equation (6)-(8), we obtain $E_0 = \left(s_{INA}^{*}, 0, 0, r_{INA}^{*}, s_{KSA}^{*}, 0\right)$ with $s_{INA}^{*} = \frac{\mu}{\theta + \mu}$, $r_{INA} = \frac{\theta}{\theta + \mu}$ and $s_{KSA}^{*} = 1$. It is called disease free equilibrium.

ii. Furthermore, if $i_{INA} \neq 0, c_{INA} \neq 0, i_{KSA} \neq 0$, then from equation (5.1) we obtain

$$s_{INA} = \frac{\mu}{B_1 i_{KSA} + \theta + \mu} \tag{9}$$

with $B_1 = \pi\beta_1 \frac{\beta}{\mu}$. If equation (6) is substituted into equation (5.2), then we obtain

$$c_{INA} = \frac{\mu B_1 i_{KSA}}{B_1 i_{KSA}(\mu + v) + B_2} \tag{10}$$

with $B_2 = (\theta + \mu)(\mu + v)$. If equation (10) is substituted into equation (5.3), then

$$i_{INA} = \frac{v\mu B_1 i_{KSA}}{B_1(\mu + v)(\mu + k\varphi) i_{KSA} + B_3} \tag{11}$$

with $B_3 = B_2(\mu + k\varphi)$. If equations (9) and (11) are substituted into equation (5.4), then we obtain

$$r_{INA} = \frac{k\varphi v B_1 i_{KSA}}{B_1(\mu + v)(\mu + k\varphi) i_{KSA} + B_3} + \frac{\theta}{B_1 i_{KSA} + \theta + \mu} \tag{12}$$

From equation (5.5), it is obtained

$$s_{KSA} = \frac{\mu}{(\theta + \mu) R_1 i_{INA} + \mu}, \tag{13}$$

with $R_1 = \frac{\tau\beta_2 \frac{A(1-\theta)}{\mu}}{\theta + \mu}$ and i_{INA} as shown in (11). Furthermore, from equations (11) and (13), it is obtained

$$i_{KSA} = \frac{R_1(\theta + \mu) v B_1 - B_3}{R_1(\theta + \mu) v B_1 + (\mu + v)(\mu + k\varphi) B_1} . \tag{14}$$

If $R_0 = \frac{v(\theta + \mu) B_1 R_1}{B_3}$ and $M = \frac{B_1}{\theta + \mu}$, then Equation (14) can be simplified as follow

$$i_{KSA} = \frac{R_0 - 1}{R_0 + M} . \tag{15}$$

Substituting Equation (15) in Equations (9) – (13) we obtain the endemic equilibrium point

$$E_1 = \left(s_{INA}^{**}, c_{INA}^{**}, i_{INA}^{**}, r_{INA}^{**}, s_{KS\_}^{**}, i_{KSA}^{**}\right)$$

where $s_{INA}^{**} = \frac{\mu(R_0+M)}{(\theta+\mu)R_0(M+1)}$, $c_{INA}^{**} = \frac{\mu B_1(R_0-1)}{B_2R_0(M+1)}$, $i_{INA}^{**} = \frac{\nu\mu B_1(R_0-1)}{B_3R_0(M+1)}$, $r_{INA}^{**} = \frac{k\varphi\nu B_1(R_0-1)}{B_3R_0(M+1)} + \frac{\theta(R_0+M)}{(\theta+\mu)R_0(M+1)}$, $s_{KSA}^{**} = \frac{M+1}{R_0+M}$, $i_{KSA}^{**} = \frac{R_0-1}{R_0+M}$. It means that the Theorem 1 has been proven. □

Basic Reproduction Number

Basic reproduction number (R_0) is the number of individuals who directly infected by an infectious person in the susceptible population (Holme & Masuda, 2015). There are many methods to find this number, for example using *M*-matrix (Raimundo, Yang, & Venturino, 2014), using spectral radius theory (Driessche & Watmough, 2002), using Jacobi and next generating method (Yang, 2014). In this paper, we use next generating matrix to determine basic reproduction number following what has done by Yang (2014).

This matrix is constructed from sub-populations that cause infection. In this model, the cause of infection is the carrier and infected population in Indonesia and the infected populations from Saudi Arabia. So, we use system (16).

$$\begin{aligned}
\frac{dc_{INA}(t)}{dt} &= \pi\beta_1 s_{INA} i_{KSA}\left(\frac{\beta}{\mu}\right) - (\mu+\nu)c_{INA} \\
\frac{di_{INA}(t)}{dt} &= \nu c_{INA} - (\mu+k\varphi)i_{INA} \\
\frac{di_{KSA}(t)}{dt} &= \tau\beta_2 s_{KSA} i_{INA}\left(\frac{A(1-\theta)}{\mu}\right) - \mu i_{KSA}
\end{aligned} \tag{16}$$

We define *F* as the linearization result of the system (16) in $E_0 = \left(\frac{\mu}{(\theta+\mu)}, 0, 0, \frac{\theta}{\theta+\mu}, 1, 0\right)$. It is a matrix of the rate new individuals infected by the disease which adds to infection class.

$$F = \begin{bmatrix} 0 & 0 & \pi\beta_1 \frac{\mu}{(\theta+\mu)}\left(\frac{\beta}{\mu}\right) \\ \nu & 0 & 0 \\ 0 & \tau\beta_2\left(\frac{A(1-\theta)}{\mu}\right) & 0 \end{bmatrix} \tag{17}$$

We also define *V* as the linearization result of the system (16) in $E_0 = \left(\frac{\mu}{(\theta+\mu)}, 0, 0, \frac{\theta}{\theta+\mu}, 1, 0\right)$. It is a matrix of the death rate and/or recovery rate which reduces infection class

$$V = \begin{bmatrix} \mu+\nu & 0 & 0 \\ 0 & \mu+k\varphi & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

The inverse matrix V is

$$V^{-1}=\begin{bmatrix} \frac{1}{\mu+v} & 0 & 0 \\ 0 & \frac{1}{\mu+k\varphi} & 0 \\ 0 & 0 & \frac{1}{\mu} \end{bmatrix} \tag{18}$$

Next generating matrix, K, is obtained as the result of multiplication between F in (17) and V^{-1} in (18)

$$K=FV^{-1}=\begin{bmatrix} 0 & 0 & \frac{\pi\beta_1\beta}{(\theta+\mu)\mu} \\ \frac{v}{\mu+v} & 0 & 0 \\ 0 & \frac{\tau\beta_2 A(1-\theta)}{\mu(\mu+k\varphi)} & 0 \end{bmatrix}$$

Basic reproduction number, $\tilde{R}_0$, is obtained from the largest eigenvalue of K in (19). We have

$$\tilde{R}_0=(R_0)^{\frac{1}{3}}$$

In this case, the basic reproduction number R_0 is taken as part of $\tilde{R}_0$.

Stability Analysis of the Equilibrium Point

2.4.1 Stability of the Disease Free Equilibrium Point

Theorem 2

i. If $R_0<1$, then the disease-free equilibrium point (E_0) is locally asymptotically stable.
ii. If $R_0>1$, then the disease-free equilibrium point (E_0) is unstable.

Proof

The Jacobian matrix of System (4) in E_0 is

$$J(E_0)=\begin{bmatrix} -(\theta+\mu) & 0 & 0 & 0 & 0 & -\pi\beta_1\frac{\mu}{\theta+\mu}\frac{\beta}{\mu} \\ 0 & -(\mu+v) & 0 & 0 & 0 & \pi\beta_1\frac{\mu}{\theta+\mu}\frac{\beta}{\mu} \\ 0 & v & -(\mu+k\varphi) & 0 & 0 & 0 \\ \theta & 0 & k\varphi & -\mu & 0 & 0 \\ 0 & 0 & -\tau\beta_2\left(\frac{A(1-\theta)}{\mu}\right) & 0 & -\mu & 0 \\ 0 & 0 & \tau\beta_2\left(\frac{A(1-\theta)}{\mu}\right) & 0 & 0 & -\mu \end{bmatrix}. \tag{20}$$

The eigenvalues of $J(E_0)$ in (20) are determined using $\det(\gamma I - J(E_0)) = 0$ where γ as eigenvalue and *I* as identity matrix, then we have the characteristic equation as follow

$$(\gamma+(\theta+\mu))(\gamma+\mu)(\gamma+\mu)\left[(\gamma+(\mu+v))(\gamma+(\mu+k\varphi))(\gamma+\mu)-v\pi\beta_1\frac{\mu}{\theta+\mu}\frac{\beta}{\mu}\tau\beta_2\left(\frac{A(1-\theta)}{\mu}\right)\right]=0 \quad (21)$$

From the equation (21), the first three of the eigenvalues are

$$\gamma_1=-(\theta+\mu),\gamma_2=-(\mu+v),\gamma_3=-(\mu+k\varphi).$$

It can be seen that γ_1, γ_2, and γ_3 are negative. Hence the rest eigenvalues satisfy

$$(\gamma+(\mu+v))(\gamma+(\mu+k\varphi))(\gamma+\mu)-v\pi\beta_1\frac{\mu}{\theta+\mu}\frac{\beta}{\mu}\tau\beta_2\left(\frac{A(1-\theta)}{\mu}\right)=0. \quad (22)$$

Equation (22) has characteristic polynomial which can be written as shown in (23)

$$\gamma^3+a_1\gamma^2+a_2\gamma+a_3, \quad (23)$$

where

$$a_1=(\mu+v)+(\mu+k\varphi)+\mu$$

$$a_2=(\mu+v)(\mu+k\varphi)+(\mu+k\varphi)\mu+(\mu+v)\mu$$

$$a_3=(\mu+v)(\mu+k\varphi)\mu[1-R_0]$$

We use Routh Hurwitz conditions (Wirkus, Swift, & Szypowski, 2017) to determine the value of each eigenvalue in (23). The eigenvalues in (23) have negative real parts if $a_1, a_2, a_3 > 0$ and $a_1a_2 > a_3$. These conditions are satisfied if $R_0 < 1$. Thus, disease free equilibrium of system (5) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$. Hence the proof completes. □

2.4.2 Stability of the Endemic Equilibrium Point

Theorem 3

i. If $R_0 > 1$, then the endemic equilibrium point (E_1) is locally asymptotically stable.

ii. If $R_0 < 1$, then the endemic equilibrium point (E_1) is unstable.

Proof

The Jacobian matrix of System (4) in E_1 is

$$J(E_1)=\begin{bmatrix} -\pi\beta_1 {i_{KSA}}^{**}\left(\frac{\beta}{\mu}\right)-(\theta+\mu) & 0 & 0 & 0 & 0 & -\pi\beta_1 {s_{INA}}^{**}\frac{\beta}{\mu} \\ \pi\beta_1 {i_{KSA}}^{**}\left(\frac{\beta}{\mu}\right) & -(\mu+v) & 0 & 0 & 0 & \pi\beta_1 {s_{INA}}^{**}\frac{\beta}{\mu} \\ 0 & v & -(\mu+k\varphi) & 0 & 0 & 0 \\ \theta & 0 & k\varphi & -\mu & 0 & 0 \\ 0 & 0 & -\tau\beta_2 & 0 & -\tau\beta_2 {i_{INA}}^{**}\frac{A(1-\theta)}{\mu}-\mu & 0 \\ 0 & 0 & \tau\beta_2 {s_{KSA}}^{**}\left(\frac{A(1-\theta)}{\mu}\right) & 0 & \tau\beta_2 {i_{INA}}^{**}\frac{A(1-\theta)}{\mu} & -\mu \end{bmatrix} \quad (24)$$

One of the eigenvalues of the $J(E_1)$ is $-\mu$ and the sign of the rest eigenvalues can be seen using Routh Hurwitz criterion. The characteristic polynomial is given by

$$\gamma^5+b_1\gamma^4+b_2\gamma^3+b_3\gamma^2+b_4\gamma+b_5, \quad (25)$$

where

$$b_1 = 2\mu + z_5 + B_1\left(\frac{R_0 - 1}{R_0 + M}\right) + \mu\left(\frac{R_0 - 1}{M + 1}\right),$$

$$b_2 = z_6 + \mu B_1\left(\frac{R_0 - 1}{R_0 + M}\right)\left(\frac{R_0 - 1}{M + 1}\right) + z_7 B_1\left(\frac{R_0 - 1}{R_0 + M}\right) + z_8\left(\frac{R_0 - 1}{M + 1}\right),$$

$$b_3 = z_9 + B_1 z_{11}\frac{R_0 - 1}{R_0 + M}\frac{R_0 - 1}{M + 1} + B_1 z_{12}\frac{R_0 - 1}{R_0 + M} + \mu z_{13}\frac{R_0 - 1}{M + 1},$$

$$b_4 = z_1 + 2\mu^2 R_1 v B_1\frac{R_0 - 1}{R_0(M + 1)} + z_2\frac{R_0 - 1}{R_0 + M}\frac{R_0 - 1}{M + 1} + z_3\frac{R_0 - 1}{R_0 + M} + z_4\frac{R_0 - 1}{M + 1},$$

$$b_5 = \frac{\mu}{\theta + \mu}\frac{R_0 - 1}{R_0 + M}\frac{R_0 - 1}{M + 1} + \mu M\frac{R_0 - 1}{R_0 + M} + \mu\frac{R_0 - 1}{M + 1} + \mu(1 - R_0),$$

$$z_1 = \mu^2(\mu + k\varphi)(\mu + \theta) + B_3 + \mu B_1,\ z_2 = \mu(\mu + k\varphi) + \frac{B_1 R_1 v}{R_0},$$

$$z_3 = \mu^2(\mu + v)B_1 + \mu B_3 M + B_3 M + \mu B_2,\ z_4 = \mu(\mu + v)B_1 + \mu B_3,$$

$$z_5 = (\mu + k\varphi) + (\mu + v) + (\mu + \theta),\ z_6 = 2\mu z_5 + \mu^2 + B_2 + (\mu + \theta)(\mu + k\varphi) + (\mu + v)(\mu + k\varphi),$$

$$z_7 = 2\mu + (\mu + k\varphi) + (\mu + v),\ z_8 = \mu^2 + \mu z_5,$$

$$z_9 = 2\mu B_2 + 2\mu(\mu + \theta)(\mu + k\varphi) + 2(\mu + v)(\mu + k\varphi) + \mu^2 z_5 + \theta(\mu + v)(\mu + k\varphi),$$

$$z_{10} = \mu(\mu + k\varphi) + \mu(\mu + v),\ z_{11} = \mu^2 + z_{10},\ z_{12} = 2z_{10} + \mu^2 + (\mu + k\varphi)(\mu + v),$$

$$z_{13} = z_9 + \mu(\mu + \theta) + (\mu + \theta)(\mu + k\varphi) + (\mu + v)(\mu + k\varphi) + B_2.$$

The Routh Hurwitz conditions that guarantee that the eigenvalues of the characteristic polynomial (25) have negative real parts are given by $b_1, b_2, b_3, b_4, b_5 > 0$ and $b_1 b_2 b_3 b_4 + 2b_1 b_4 b_5 + b_2 b_3 b_5 > b_5^2 + b_1 b_2^2 b_5 + b_1^2 b_4^2 + b_3^2 b_4$. These conditions are easily seen to be satisfied if $R_0 > 1$. Thus, the endemic equilibrium point (E_1) of system (5) is locally asymptotically stable when $R_0 > 1$ and unstable when $R_0 < 1$. This completes the proof. □

RESULT AND DISCUSSION

The mathematical model of the *Meningococcal meningitis* disease spread would be simulated to provide a geometric pattern in accordance with the conditions of the basic reproduction number and the influence of giving vaccine to the Hajj / Umrah pilgrims from Indonesia. Basic reproduction number can be used to determine whether the disease disappears or endemic in the population.

The average number of Umrah pilgrims from Indonesia is 195 people per day, hence there are 1,365 people per week to Umrah. The average number of Hajj pilgrims from Indonesia is 154,000 people per year, hence there are 2,961 Hajj pilgrims per week (Ministry of Religious Affairs of the Republic of Indonesia, 2015). Therefore the number of prospective Hajj/Umrah pilgrims from Indonesia per week (A) is 4,326 people. On the other side, the average number of Hajj/Umrah pilgrims from Saudi Arabia is 700,000 people per year (General Authority for Statistic, 2017), so every week there is (β) 13,461 people from Saudi Arabia.

Table 2. The parameter values of the *Meingococcal meningitis* model

Parameter	Parameter value	Reference
μ	0.000274	Roser (2019)
π	0.00023	Assumption
τ	0.0000742	Assumption
β_1	0.7	Martinez, M. J. F. (2013)
β_2	0.5	Assumption
φ	0.1	Assumption
k	1	Stephens, *et all* (2007)
v	1	Irving, *et all* (2012)
θ	$0 \le \theta \le 1$	Assumption

1. Mathematical Model Simulation on *Meingococcal meningitis* Infectious Disease Spread without Vaccination Effect or $\theta = 0$

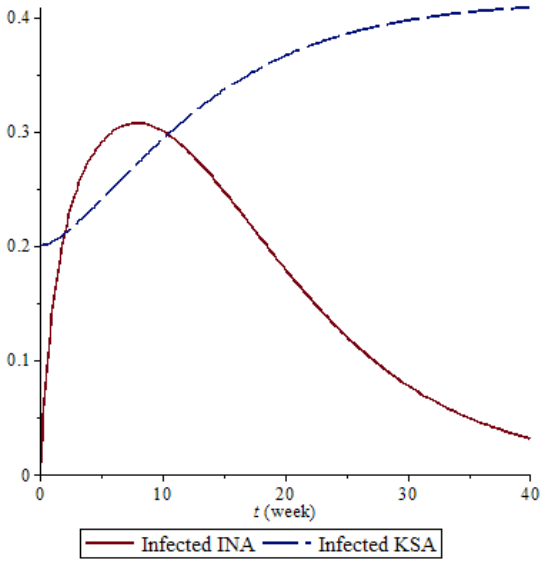

Figure 2. Simulation with $\theta = 0$

In Figure 2, for $\theta = 0$, it can be seen that if vaccination is not given on population from Indonesia, then both i_{INA} and i_{KSA} population increase significantly and the disease will continue to spread. Under these circumstances, the value of R_0 is $2874.593 > 1$ and it only need 10 weeks to approach the highest i_{INA} population.

Mathematical Model Simulation on *Meingococcal meningitis* Infectious Disease Spread with Vaccination Influence for $\theta = 0.5$

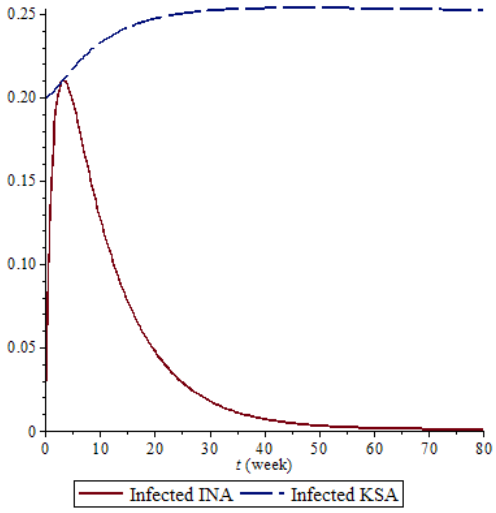

Figure 3. Simulation with $\theta = 0.5$

In Figure 3, with a vaccination rate of $\theta = 0.5$, it appears that the number of infected individuals present in Indonesia appears to decrease more rapidly than in Figure 2. This indicates that the higher vaccinations are given, the infected individuals will disappear faster than the population, but in Figure 3 this still has not led to zero so that at some time *Meningococcal meningitis* disease will remain in the population indefinitely. Under these circumstances, the condition $R_0 = 0.787207 < 1$. That is, infected people can hardly infect others but infected people still exist in the population.

2. Mathematical Model Simulation on *Meingococcal meningitis* Infectious Disease Spread with Vaccination Influence for $\theta = 0.9$

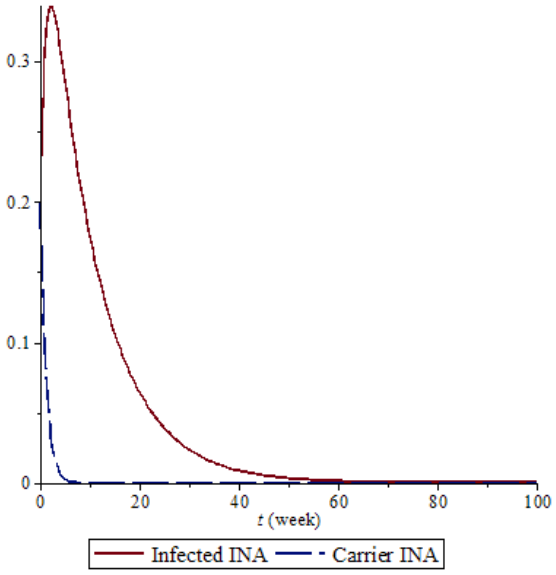

Figure 4. Simulation with $\theta = 0.9$

In Figure 4, it can be seen that many infected and carrier individuals present in Indonesia decrease to zero so that at certain times *Meningococcal meningitis* disease disappears from the population. This suggests that the higher vaccination is given to susceptible individuals present in Indonesia, then the disease will disappear faster than the population. Vaccination with $\theta = 0.9$ is considered the most effective because carriers and infected populations can lead to zero. On the graph, it appears that the disease will disappear at week 50. In these circumstances, the condition $R_0 = 0.087489 < 1$ which means the infected population will not infect other populations.

CONCLUSIONS

Based on the discussion that has been described above it can be concluded that mathematical modeling of infectious disease of Meningococcal meningitis spread in the form of nonlinear differential equations systems $\boldsymbol{SCIR - SI}$ models were shown in the system (1) and its transformation was in the system (4). The disease-free equilibrium point was obtained

$$\boldsymbol{E}_0 = \left(\boldsymbol{s_{INA}}^*, 0, 0, \boldsymbol{r_{INA}}^*, \boldsymbol{s_{KSA}}^*, 0\right)$$

where $s_{INA}^* = \dfrac{\mu}{\theta+\mu}, \boldsymbol{r_{INA}}^* = \dfrac{\theta}{\theta+\mu}$ and $s_{KSA}^* = 1$

and the endemic equilibrium point was obtained

$$E_1 = \left(s_{INA}^{**}, c_{INA}^{**}, i_{INA}^{**}, r_{INA}^{**}, s_{KSA}^{**}, i_{KSA}^{**}\right)$$

where $s_{INA}^{**} = \dfrac{\mu\left(R_0+M\right)}{\left(\theta+\mu\right)R_0\left(M+1\right)}$, $c_{INA}^{**} = \dfrac{\mu B_1\left(R_0-1\right)}{B_2 R_0\left(M+1\right)}$, $i_{INA}^{**} = \dfrac{\nu\mu B_1\left(R_0-1\right)}{B_3 R_0\left(M+1\right)}$,

$r_{INA}^{**} = \dfrac{k\varphi\nu B_1\left(R_0-1\right)}{B_3 R_0\left(M+1\right)} + \dfrac{\theta\left(R_0+M\right)}{\left(\theta+\mu\right)R_0\left(M+1\right)}$, $s_{KSA}^{**} = \dfrac{M+1}{R_0+M}$, $i_{KSA}^{**} = \dfrac{R_0-1}{R_0+M}$.

If $R_0 < 1$, then the disease-free equilibrium point is local asymptotically stable and if $R_0 > 1$, then the disease-free equilibrium point is unstable. If $R_0 > 1$, then the endemic equilibrium point is local asymptotically stable and if $R_0 < 1$, then the endemic equilibrium point is unstable. Based on the simulation, with the influence of giving a vaccine for Hajj/Umrah from Indonesia, it showed that the higher the level of vaccinations given, it will cause the decreasing number of infected individuals. This suggests that vaccination programs can be used to reduce the number of infected individuals caused by the spread of infectious diseases of *Meningococcal meningitis* that occur between two different areas. In this paper we only discuss a model of *Meningococcal meningitis* infectious diseases spread with vaccination effect only from Indonesian susceptible populations that are given the vaccination. Therefore, further research is recommended to discuss the spread of infectious diseases of *Meningococcal meningitis* by the influence of vaccinations for susceptible populations from Indonesia and Saudi Arabia. Then for stability analysis it is suggested to discuss the global stability analysis for the spread of infectious disease of *Meningococcal meningitis* with or without the vaccination.

REFERENCES

Asamoah, J. K., Nyabadza, F., Seidu, B., Chand, M., & Dutta, H. (2018). Mathematical modelling of bacterial meningitis transmission dynamics with control measures. *Computational and Mathematical Methods in Medicine, 2018*, 1-21.

Blyuss, K. B. (2016). Mathematical modelling of the dynamics of meningococcal meningitis in Afric. In P. Aston, A. Mulholland, & K. Tant, *UK Success Stories in Industrial Mathematics* (pp. 221–226). Springer, Cham.

Centers for Disease Control and Prevention. (2015). Retrieved March 20, 2019, from https://www.cdc.gov/vaccines/pubs/pinkbook/downloads/mening.pdf

Driessche, P. v., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences, 180*, pp 29-48.

General Authority for Statistic. (2017). *Hajj Statistics 1438 H.* Saudy Arabia.

Holme, P., & Masuda, N. (2015). The basic reproduction number as a predictor for epidemic outbreaks in temporal networks. *PLoS ONE, 10*(3), e0120567.

Hung, M.-C., & Christodoulides, M. (2013). The biology of neisseria adhesins. *Biology, 2*, pp 1054-1109.

Irving, T., Blyuss, K., Coljin, C., & Trotter, C. (2012). Modelling meningococcal meningitis in the African meningitis belt. *Epidemiology&Infection, 140*, pp 897-905.

Martcheva, M., & Crispino-O'Connell, G. (2003). The transmission of meningococcal infection: A mathematical study. *Journal of Mathematical Analysis and Applications, 283*, pp 251-275.

Martınez, M. F., Merino, E. G., Sanchez, E. G., Sanchez, J. G., Reyc, A. M., & Sanchezd, G. R. (2013). A mathematical model to study the meningococcal meningitis. *International Conference on Computational Science* (pp. 2492 – 2495). Elsevier.

Ministry of Religious Affairs of the Republic of Indonesia. (2015, Mei 11). Retrieved March 21, 2019, from https://haji.kemenag.go.id/v3/content/rata-rata-jemaah-umrah-berangkat-perhari-195-orang

Perko, L. (2001). *Differential equations and dynamical systems.* New York: Springer.

Raimundo, S. M., Yang, H. M., & Venturino, E. (2014, August). Theoretical sssessment of the relative incidences of sensitive and resistant tuberculosis epidemic in presence of drug treatment. *Mathematical Biosciences and Engineering, 11*(4), pp 971–993.

Roser, M. (2019). *Life Expectancy.* Retrieved April 29, 2019, from OurWorldInData.org: https://ourworldindata.org/life-expectancy

Stephens, D. S., Greenwood, B., & Brandtzaeg, P. (2007). Epidemic meningitis, meningococcaemia,

and Neisseria meningitidis. *Lancet, 369*, pp 2196-2210.

Vuocolo, S., Balmer, P., Gruber, W. C., Jansen, K. U., Anderson, A. S., Perez, J. L., et al. (2018). Vaccination strategies for the prevention of meningococcal disease. *Human Vaccines & Immunotherapeutics, 14*(5), pp 1203–1215.

Wiah, E., & Adetune, I. (2010). A mathematical model of cerebrospinal meningitis epidemic: A case study for Jirapa District, Ghana. *KMITL Science and Technology Journal, 10*(2), pp 63-73.

Wirkus, S. A., Swift, R. J., & Szypowski, R. S. (2017). *A Course in Differential Equations with Boundary Value Problems, Second Edition.* New York: CRC Press.

World Health Organization. (2002). *Control of epidemic meningococcal disease. WHO practical guidelines 2nd edition.* WHO/EMC/BAC/98.3.

Yang, H. M. (2014). The basic reproduction number obtained from Jacobian and next generation matrices–A case study of dengue transmission modelling. *BioSystems, 126*, pp 52-75.

Yezli, S., Assiri, A. M., Alhakeem, R. F., Turkistani, A., & Alotaibi, B. (2016). Meningococcal disease during the Hajj and Umrah mass gatherings. *International Journal of Infectious Diseases, 47*, 60-64.