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ANALYTIC STUDY OF THERMOHALINE CONVECTIVE STABILITY IN A COUPLE-STRESS FLUID

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Abstract: This study examines stability by conducting a nonlinear stability analysis on the thermohaline flow of a steady, viscous, incompressible couple-stress fluid, utilizing a generalized energy method. It is observed that the linear and nonlinear thresholds are the same, and the physics of the onset of convection is fully captured. The couple-stress and solute gradient parameters are found to have a stabilizing effect on the system.

Keywords: *Stability, energy method, Couple-stress fluid, thermohaline convection, Rayleigh number.*

1. Introduction

Hydrodynamic stability theory has been the subject of intensive study for decades, as it predominantly concerns the identification of critical Rayleigh number values that depict the stability region [see Joseph (1965, 1966); Straughan (2004)]. While linear theory does not guarantee stability, it does establish conditions for the instability of hydrodynamic systems. Under certain conditions, the energy technique of nonlinear theory ensures the stability of hydrodynamic systems but cannot conclusively prove instability. Thus, the use of a nonlinear approach to study the effects of finite perturbations on flow becomes vital. Initial credit for the energy method goes to Reynolds (1895) and Orr (1907), but it was later improved by Serrin (1959) and Joseph (1965, 1966, 1976). This classical energy technique, successful in many problems [Rionero, 1968; Galdi, 1985; Galdi & Straughan, 1985], has been confronted in many situations. Later, numerous authors (Galdi & Padula, 1990; Straughan, 2004; Rionero & Mulone, 1988; Mulone & Rionero, 1989) employed and improved the classical energy theory, and its generalization is now considered more successful in analyzing many complex theories.

Industrial and technological applications of couple-stress fluids, which include pumping fluids like synovial joint fluid, synthetic fluids, liquid crystals, animal blood, and the theory of lubrication, have attracted researchers to study the properties and behaviors of such fluids. The mathematical relation for Couple-stress fluid flow, as proposed by Stokes (1966), has distinct characteristics like couple stress forces, body couples, and non-symmetric stress tensors. Stokes (1984) provides an excellent description of this theory. Sunil et al. (2013, 2014, 2019) examined the stability problem of couple-stress fluid using the energy method. It is observed that the critical thermal Rayleigh values for linear instability and nonlinear stability coincide, indicating that subcritical instabilities are not present. It is known that saline can easily adhere or suspend in any fluid, and in certain cases, like animal blood, saline is present in the fluid. It is, therefore, important to consider this aspect when studying the stability of couple-stress liquids.

This paper aims to discuss the thermohaline convective stability in couple-stress fluids and analyze the impact of the presence of couple stress and solute concentration on convection, utilizing both linear and nonlinear methods of stability. The calculated critical Rayleigh number values depict the onset of convection and also provide an estimate of the stability region. These estimates may be useful for experiments to control heat transfer in such liquids. This paper identifies research opportunities and challenges for future research and has, to the best of our knowledge, not appeared in the literature thus far.

2. Mathematical Model

Consider a layer of couple stress fluid between two parallel plates d distance apart, extending infinitely in the horizontal direction, with constant viscosity. The temperature $T_{a,v}$ at the lower and upper surfaces $z = d/2$ and $-d/2$ is considered to be

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fixed as T_l, T_u, respectively. The temperature gradient $\beta_T \left(= \left|\frac{dT}{dz}\right|\right)$ is maintained across the layer. The fluid layer heated and soluted from below confines with stress free boundaries and a gravitational force acting vertically downward along the z-direction.

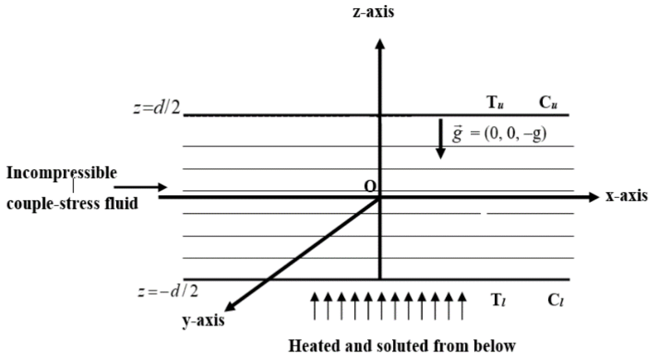

Figure 1. Pictorial representation of the mathematical model.

The governing equations of the convective system of the fluid with couple stress forces (by using Boussinesq approximation) are (Sunil et al., 2011; Choudhary & Sunil, 2019):

$$\nabla \cdot \vec{q}_s = 0 \tag{1}$$

$$\rho_r \left(\frac{\partial}{\partial t} + \vec{q}_s \cdot \nabla\right) \vec{q}_s = -\nabla p_1 + \rho_r[1 - \alpha(T - T_{a\,v}) + \alpha'(C - C_{a\,v})]\vec{g} + (\mu - \mu'\nabla^2)\nabla^2 \vec{q}_s \tag{2}$$

$$\frac{\partial T}{\partial t} + \vec{q}_s \cdot \nabla T = \kappa \nabla^2 T, \tag{3}$$

$$\frac{\partial C}{\partial t} + \vec{q}_s \cdot \nabla C = \kappa' \nabla^2 C. \tag{4}$$

Here, ρ_f is the fluid density ρ_r is the reference density, $\vec{q}_s$ is the fluid velocity, $\vec{g}$ is the acceleration due to gravity, t is the time, p_1 is the pressure field, μ is the coefficient of viscosity, μ' is the coefficient of visco-elasticity, κ is the thermal diffusivity, κ' is the solute diffusivity, α is the thermal expansion coefficient and $\alpha'$is the solute expansion coefficient. The solute concentrations $C_{a\,v}$ at the lower and upper planes are fixed as C_l, C_u, respectively, while maintaining the solute gradient $\beta_C \left(= \left|\frac{dC}{dz}\right|\right)$ across the layer. The basic state ('b') is quiescent, and is given by:

$$\vec{q}_s = \vec{0}, \quad p_1 = p_{1b}(z), \quad \rho_f = \rho_{f\,b}(z) = \rho_r(1 + \alpha\beta_T z - \alpha'\beta_C z), \quad T = T_b(z) = -\beta_T z + T_{a\,v}, C = C_b(z) = -\beta_C z + C_{a\,v}, \quad \beta_T = \frac{T_l - T_u}{d}, \quad \beta_C = \frac{C_l - C_u}{d}, \tag{5}$$

To perform stability analysis, let us introduce perturbations $\vec{q}_s', p_1', \rho', \theta$and γ representing velocity, pressure, density, temperature and concentration, respectively, to the basic state. The perturbations equations are:

$$\rho_0 \frac{\partial \vec{q}_s{}'}{\partial t} + \rho_r \vec{q}_s{}' \cdot \nabla \vec{q}_s{}' = -\nabla p_1' + \mu \nabla^2 \vec{q}_s{}' - \mu' \nabla^4 \vec{q}_s{}' + \rho_r g(\alpha\theta - \alpha'\gamma)\boldsymbol{k}, \tag{6}$$

$$\nabla \cdot \vec{q}_s{}' = 0, \tag{7}$$

$$\frac{\partial \theta}{\partial t} + \vec{q}_s{}' \cdot \nabla \theta = \kappa \nabla^2 \theta + \beta_T w \tag{8}$$

$$\frac{\partial \gamma}{\partial t} + \vec{q}_s{}' \cdot \nabla \gamma = \kappa' \nabla^2 \gamma + \beta_C w. \tag{9}$$

Boundary conditions (BC's) on $\vec{q}_s', \theta, \gamma$ which satisfy a plane tilling periodicity are:

$$\vec{q}_s' = 0, \quad \theta = 0, \quad \gamma = 0 \text{ at } z = \pm\frac{d}{2} \tag{10}$$

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3. Stability Analysis by Generalized Energy Method

To analyze stability, the perturbation equations (6) – (9) are written in non-dimensional form (dropping *) as follows:

$$\frac{\partial \vec{q}_s}{\partial t} + \vec{q}_s \cdot \nabla \vec{q}_s = -\nabla p_1 + \nabla^2 \vec{q}_s - F_C \nabla^4 \vec{q}_s + R^{\frac{1}{2}} \theta \boldsymbol{k} - \frac{S^{\frac{1}{2}}}{Le} \gamma \boldsymbol{k} \tag{11}$$

$$\nabla \cdot \vec{q}_s = 0, \tag{12}$$

$$\frac{\partial \theta}{\partial t} + \vec{q}_s \cdot \nabla \theta = \nabla^2 \theta + R^{\frac{1}{2}} w, \tag{13}$$

$$\frac{\partial \gamma}{\partial t} + \vec{q}_s \cdot \nabla \gamma = \frac{1}{Le} \nabla^2 \gamma + S^{\frac{1}{2}} w \tag{14}$$

The below mentioned dimensionless quantities and parameters are used for non dimensionalizing the perturbed equations

$$t^* = \frac{\mu}{\rho_0 d^2} t, \quad \vec{q}^*{}_s = \frac{d}{\nu} \vec{q}_s', \quad p_1{}^* = \frac{d^2}{\rho_r \nu^2} p_1', \quad \theta^* = \frac{R^{\frac{1}{2}}}{\beta_T d} \theta, \quad \gamma^* = \frac{S^{\frac{1}{2}}}{\beta_C d} \gamma, z^* = \frac{1}{d} z, R = \frac{g \alpha \beta_T \rho_r d^4}{\mu \kappa}, \quad S = \frac{g \alpha' \beta_C \rho_r d^4}{\mu \kappa'}, Le = \frac{\kappa}{\kappa'} \text{ and } F_C = \frac{1}{\nu} \frac{\mu'}{\rho_r d^2} \tag{15}$$

Here, R is the thermal Rayleigh number, S is the solute Rayleigh number, F_c is the couple-stress parameter and Le is the Lewis number. Multiplying equations (11) by $\vec{q}_s$ (13) by θ, (14) by γ and integration over V and utilizing the given conditions, we obtain:

$$\frac{1}{2} \frac{d \|\vec{q}_s\|^2}{dt} = -\|\nabla \vec{q}_s\|^2 - F_c \|\nabla^2 \vec{q}_s\|^2 + R^{\frac{1}{2}} \langle w\theta \rangle - S^{\frac{1}{2}} \langle w\gamma \rangle, \tag{16}$$

$$\frac{1}{2} \frac{d \|\theta\|^2}{dt} = -\|\nabla \theta\|^2 + R^{\frac{1}{2}} \langle w\theta \rangle, \tag{17}$$

$$\frac{1}{2} \frac{d \|\gamma\|^2}{dt} = -\frac{1}{Le} \|\nabla \gamma\|^2 + S^{\frac{1}{2}} \langle w\gamma \rangle \tag{18}$$

Here, the symbol $\langle \cdot \rangle$ is for integration over $V$and $\|\cdot\|$ is the $L^2(V)$ norm.

From Eq. (16) – (17), an L^2energy $E_0(t)$ is constructed and the change of $E_0(t)$ is

$$\frac{dE_0}{dt} = I_a - D_a, \tag{19}$$

where

$$E_0 = \frac{1}{2} \|\theta\|^2 + \frac{\lambda_1}{2} \|\vec{q}_s\|^2 - \frac{\lambda_2}{2} \|\gamma\|^2, \tag{20}$$

$$I_a = (1 + \lambda_1) R^{\frac{1}{2}} \langle w\theta \rangle - \lambda_1 \frac{S^{\frac{1}{2}}}{Le} \langle w\gamma \rangle - \lambda_2 S^{\frac{1}{2}} \langle w\gamma \rangle, \tag{21}$$

$$D_a = \|\nabla \theta\|^2 + \lambda_1 \|\nabla \vec{q}_s\|^2 + \lambda_1 F_c \|\nabla^2 \vec{q}_s\|^2 - \frac{\lambda_2}{Le} \|\nabla \gamma\|^2, \tag{22}$$

with λ_1 and λ_2 being positive coupling parameters.

In equation (20), the term $\frac{\lambda_2}{2} \|\gamma\|^2$ has a negative sign, which indicates that the system consumes energy due to the solute concentration. The energy consumed by the solute is less than the energy produced by the velocity and temperature. The energy dissipated by the solute is less than the energy dissipated by these two factors. This always ensures that all the RHS terms of equations (20) and (22) are less than the LHS terms.

We define

$$m_{max} = \max_H \frac{I_a}{D_a} \tag{23}$$

where H is the space of admissible solutions, and take m_{max}, so that

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$$\frac{dE_0}{dt} \le -e_0 D_a \tag{24}$$

with $e_0 = 1 - m_{max}$.

Using Poincare inequality, it is established that

$$D_a \ge \pi^2 \left(\|\theta\|^2 + \lambda_1(1+\pi^2 F_c)\|\vec{q}_s\|^2 - \frac{\lambda_2}{Le}\|\gamma\|^2 \right) \ge E_0 \tag{25}$$

Eq. (25) is hold if $\frac{\lambda_2}{Le}\|\gamma\|^2 < \lambda_1(1+\pi^2 F_c)\|\vec{q}_s\|^2$.

Using the (24) and (25), one finds

$\frac{dE_0}{dt} \le -e_0 E_0$,

and by integration of this between limits 0 and t, the energy estimate is

$$E_0(t) \le exp(-e_0 t)E_0(0). \tag{26}$$

Thus, the exponential fast decay of$E$ assures conditional stability for all $E_0(0)$.

Variational Problem

Equation (23) is solved by using calculus of variation to maximize at the critical value m_{max}. By using the transformations $\hat{\vec{q}}_s = \sqrt{\lambda_1}\vec{q}_s$ and $\hat{\gamma} = \sqrt{\lambda_2}\gamma$, the Euler-Lagrange equations are given as (by dropping caps)

$$-2F_c\nabla^4\vec{q}_s + 2\nabla^2\vec{q}_s + R^{\frac{1}{2}}(1+\lambda_1)\frac{1}{\lambda_2^{\frac{1}{2}}}\theta\hat{\boldsymbol{k}} - S^{1/2}(1+\lambda_2)\frac{1}{\lambda_1^{\frac{1}{2}}\lambda_2^{\frac{1}{2}}}\gamma\hat{\boldsymbol{k}} = 2\nabla\eta, \tag{27}$$

$$2\nabla^2\theta + R^{1/2}(1+\lambda_1)\frac{1}{\lambda_1^{\frac{1}{2}}}w = 0 \tag{28}$$

$$\frac{2}{Le}\nabla^2\gamma + S^{\frac{1}{2}}\left(\lambda_2 + \frac{\lambda_1}{Le}\right)\frac{1}{\lambda_1^{\frac{1}{2}}\lambda_2^{\frac{1}{2}}}w = 0 \tag{29}$$

where η is introduced as Lagrange's multiplier, due to the solenoidal property of $\vec{q}_s$.

The third component of curlcurl of equation (27) is written as

$$-2F\nabla^6 w + 2\nabla^4 w + R^{\frac{1}{2}}(1+\lambda_1)\frac{1}{\lambda_1^{\frac{1}{2}}}\nabla_1^2\theta - S^{1/2}\left(\lambda_2 + \frac{\lambda_1}{Le}\right)\frac{1}{\lambda_1^{\frac{1}{2}}\lambda_2^{\frac{1}{2}}}\nabla_1^2\gamma = 0 \tag{30}$$

Now, consider a plane tiling solution

$$(w, \theta, \gamma) = [W(z), \Theta(z), \Gamma(z)]g(x,y) \tag{31}$$

Here, $\nabla_1^2 g + a^2 g = 0$, where 'a' is the non-zero wave number (Straughan, 2001; Chandrasekhar, 1981). Thus, the equations (28)-(30) are represented as

$$-2F_c(D^2-a^2)^3W + 2(D^2-a^2)^2W - \frac{R^{\frac{1}{2}}a^2}{\lambda_1^{\frac{1}{2}}}(1+\lambda_1)\Theta + \frac{S^{\frac{1}{2}}a^2}{\lambda_1^{\frac{1}{2}}\lambda_2^{\frac{1}{2}}}\left(\lambda_2 + \frac{\lambda_1}{Le}\right)\Gamma = 0, \tag{32}$$

$$2(D^2-a^2)\Theta + \frac{R^{1/2}}{\lambda_1^{\frac{1}{2}}}(1+\lambda_1)W = 0 \tag{33}$$

$$\frac{2}{Le}(D^2-a^2)\Gamma + \frac{S^{\frac{1}{2}}}{\lambda_1^{\frac{1}{2}}\lambda_2^{\frac{1}{2}}}\left(\lambda_2 + \frac{\lambda_1}{Le}\right)W = 0 \tag{34}$$

and the BC's are

$$W = 0, \quad D^2W = 0, \quad D^4W = 0, \quad \Theta = 0, \quad \Gamma = 0 \quad \text{at } z = \pm\frac{1}{2} \tag{35}$$

The functions W, Θ and Γ, satisfying (35) are given by

$$W = l_0 \cos\pi z, \Theta = m_0 \cos\pi z \Gamma = n_0 \cos\pi z, \tag{36}$$

where l_0, m_0 and n_0 are constants. Using solution (36) in (32)-(34), the polynomial equations with coefficients of l_0, m_0 and n_0 are obtained. Condition for existence of nontrivial solution of these equations' yields

$$R_e = \frac{4(1+x)^3\{1+F_1(1+x)\}}{x\frac{1}{\lambda_1}(1+\lambda_1)^2} + \frac{S_1 Le\left(\lambda_2 + \frac{\lambda_1}{Le}\right)^2}{\lambda_2(1+\lambda_1)^2} \tag{37}$$

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where $R_e = \frac{R}{\pi^4}$, $S_1 = \frac{S}{\pi^4}$, $x = \frac{a^2}{\pi^2}$, $F_1 = \pi^2 F_c$.

The optimal value of λ_1 and λ_2 is obtained from the conditions $\frac{dR_e}{d\lambda_1} = 0$ and $\frac{dR_e}{d\lambda_2} = 0$, respectively, and are found to be

$$\lambda_1 = 1 \text{ and } \lambda_2 = \frac{1}{Le}. \quad (38)$$

Using (38) in equation (37), the Rayleigh number becomes

$$R_e = \frac{(1+x)^3[1+F_1(1+x)]}{x} + S_1. \quad (39)$$

R_e attains minimum when

$$3F_1x^4 + 2(1+4F_1)x^3 + 3(1+2F_1)x^2 - (1+F_1) = 0. \quad (40)$$

From the condition $\frac{dR_e}{dx} = 0$, the critical wave number values are derived numerically using the Newton-Raphson method. From equation (39), the required critical thermal Rayleigh number R_{ce} is obtained.

It is thus crucial to perform a standard normal mode technique on the perturbed equations (6) − (9), without including nonlinear terms, to investigate linear instability and determine their solution in the form (31) for the comparison of nonlinear results. Using the boundary conditions (35), the thermal Rayleigh number is obtained as

$$R_\ell = \frac{(1+x)^3[1+F_1(1+x)]}{x} + S_1 = R_e. \quad (41)$$

Equation (35) simplifies to $R_\ell = \frac{(1+x)^3}{x} = R_e$, in the absence solute gradient ($S_1 = 0$) and of couple stress parameter ($F_1 = 0$), i.e., the linear instability boundary ≡ the nonlinear stability boundary, indicating no sub-critical instabilities region exists.

Table 1. Dependence of the R_c on (F_1).

F_1	x_c	R_c
0	0.5	106.75
1	0.387	116.46
2	0.366	125.99
3	0.357	135.49
4	0.351	144.99
5	0.348	154.48
6	0.346	163.97
7	0.344	173.45
8	0.343	182.94
9	0.342	192.42
10	0.341	201.90

Table 2. Dependence of the R_c on S_1, for various values of F_1.

S_1	R_c $\begin{pmatrix} F_1 = 1 \\ x_c = 0.387 \end{pmatrix}$	R_c $\begin{pmatrix} F_1 = 5 \\ x_c = 0.348 \end{pmatrix}$	R_c $\begin{pmatrix} F_1 = 9 \\ x_c = 0.342 \end{pmatrix}$
100	116.46	154.48	192.42
200	216.46	254.48	292.42
300	316.46	354.48	392.42
400	416.46	454.48	492.42
500	516.46	554.48	592.42
600	616.46	654.48	692.42
700	716.46	754.48	792.42
800	816.46	854.48	892.42
900	916.46	954.48	992.42

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4. Results and Discussion

The critical wave number $x_c = x_{ce} = x_{c\ell}$ and critical thermal Rayleigh number $R_c = R_{ce} = R_{c\ell}$ were a function of couple stress parameter F_1 and solute gradient S_1. The variations given in Tables 1 and 2 are illustrated graphically in Figures 2 and 3.

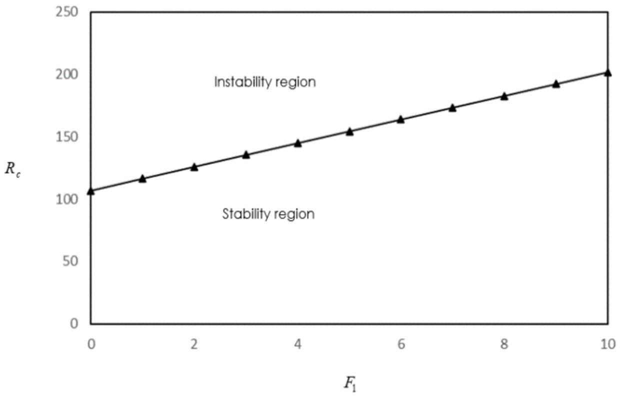

Figure 2. Plot of $R_c$versus F_1 for S_1=100.

From figure 2, it is found that the convection is advanced due to couple-stress parameter F_1, as R_c increases with an increase in F_1. Therefore, this parameter attempts to stabilize the convection in fluid. Additionally, the linear and nonlinear Rayleigh numbers are found to be the same. Table 1 shows that the couple-stress fluid stabilizes thermally more as compared to the standard fluid.

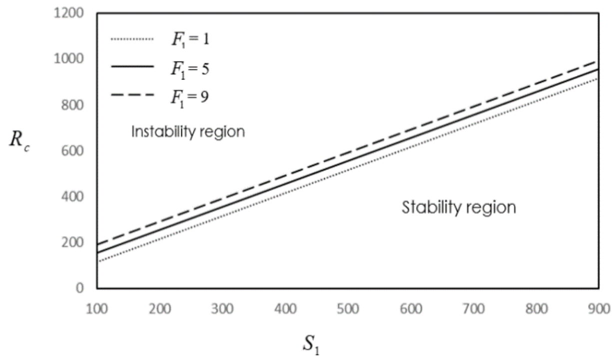

FIGURE 3. Plot of $R_c$versus S_1 for different values of F_1.

The variation of $R_c$with variation in S_1 for various values of F_1 is given in Table 2 and Figure 3. This clearly demonstrates the stabilizing effect of the solute parameter S_1 because R_c increases with an increase in S_1.

5. Conclusion

The results obtained using the generalized energy technique for the nonlinear system are the same as those obtained by linear theory, indicating that the boundaries of stability and instability coincide. There is no possibility of the existence of any subcritical instability. This finding is significant because it suggests that by controlling the parameter values, the fluid can be stabilized, and heat transfer can also be controlled. An increase in the values of parameters shows that the couple stresses and the presence of solute stabilize the system. Consequently, couple stress fluids are observed to be more stable than ordinary fluids. Therefore, in applications where higher stability requirements exist, such fluids can be utilized.

The study could be enhanced by including the effects under rigid type boundary conditions and also by considering viscosity as a function of temperature and pressure.

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