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An Integrated Production System for a Single Installment Policy of Raw Material

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ABSTRACT This paper considers a manufacturing system which procures raw materials from suppliers and processes them to make a finished product. The problem is to determine an optimal production size with a single installment of raw materials to satisfy a deterministic time-varying demand process by minimizing the total relevant cost. We developed a mathematical model for the problem and then compared the result with a lot-for-lot model with single installment and lot-for-lot model with multiple installments. From the optimality condition, we derived an optimal solution procedure for the proposed model. We present numerical examples for a discussion and comparison.

ABSTRAK Kertas kerja ini mempertimbangkan sistem pembuatan dimana bahan mentah diperolehi dari pembekal dan kemudiannya diproses kepada bahan siap. Masalahnya adalah untuk menentukan saiz pengeluaran dengan satu pesanan bagi memenuhi proses permintaan yang berketentuan berubah dengan masa dengan meminimakan jumlah kos yang berkaitan. Model matematik dibina untuk masalah tersebut dan keputusannya dibandingkan dengan model lot demi lot dengan satu pesanan dan lot demi lot dengan beberapa pesanan. Daripada syarat pengoptimumam, prosedur penyelesaian optimal diterbitkan untuk model yang dicadangkan. Beberapa contoh berangka diberikan untuk perbincangan dan perbandingan.

(Batch size, manufacturing system, deterministic time-varying demand, optimality condition)

INTRODUCTION

In many manufacturing systems, the quantity of raw materials needed for production is dependent on the production size. Therefore, it is preferable to unify the optimization of both elements under a single model. Khan and Sarker [1], Sarker and Newton [8], and Sarker and Parija [6] developed a few models for this system under continuous supply and a constant demand rate. In reality, this assumption is very restrictive especially during the growth and decline phases of the product life cycle, it is either increasing or decreasing with time. Omar and Smith [2] have developed a lot-for-lot model for this system under linearly increasing time-varying demand process. Omar and Supadi [3] have extended this model and developed a lot-for-lot model with multiple equal installments of raw material for each production lot size.

In this paper, we consider a case where the whole requirement of raw material during the planning horizon will be ordered with a single installment at the beginning of the planning horizon. This model is more appropriate in a case where an ordering cost is more expensive compare to the holding cost. In this model we determine the number of production batches, n, and the manufacturing quantity for each batch which give the minimum total cost.

By using the optimality condition (see Omar and Yeo [4]), we derived an iterative optimal solution procedures. We find an optimal solution by using Microsoft Excel Solver. Finally we presented numerical examples for discussion and comparison.

MATHEMATICAL FORMULATION

The cost factors which are considered here are the raw material ordering cost, the manufacturing set-up cost, the raw material holding cost and the finished product holding cost. Here, we state the assumptions and notations.

Assumptions

1. The supply of raw material and finished product are continuous.
2. No shortages are permitted.
3. A single product inventory system is considered over a known and finite planning horizon, H.
4. During production time, finished product becomes immediately available to meet the demand process.
5. The demand rate of finished product at time t in (0, H) is $f(t) = a + bt$.
6. The finite production rate is P units per unit time and $P > f(t)$ for all t.
7. For simplicity we only consider one type of raw material (j = 1) is required to produce one unit of a product ($r_1 = 1$).
8. The inventory level of the finished product is zero during the start and the end of each cycle.

Notations

1. c_p is the fixed manufacturing set-up cost.
2. c_1 is the ordering cost for raw material 1.
3. h_p is the carrying inventory cost per unit per unit time for finished products.
4. h_1 is the carrying inventory cost per unit per unit time for raw material 1.
5. n is the total number of batch replenishment (t_n = H).
6. r_1 is the amount/quantity of raw material 1 required in producing one unit of a product.

Figure 1 gives a graphical representation of the model when $n = 3$. It shows the inventory level of raw material and finished product against time.

In this model, raw material will be ordered once at the beginning of the planning horizon for the whole batches in the production planning. This case is more suitable when the ordering cost of raw material is expensive comparing to the ordering cost.

We assume that production starts at time t_0 until t_0* and as soon as the previous batch has been used up at time t_i until t_i* where $i = 1, 2, \ldots, n - 1$. The accumulated inventory during the production up-time is used for making delivery during the production down-time until the inventory is exhausted. The production is then resumed and the cycle is repeated.

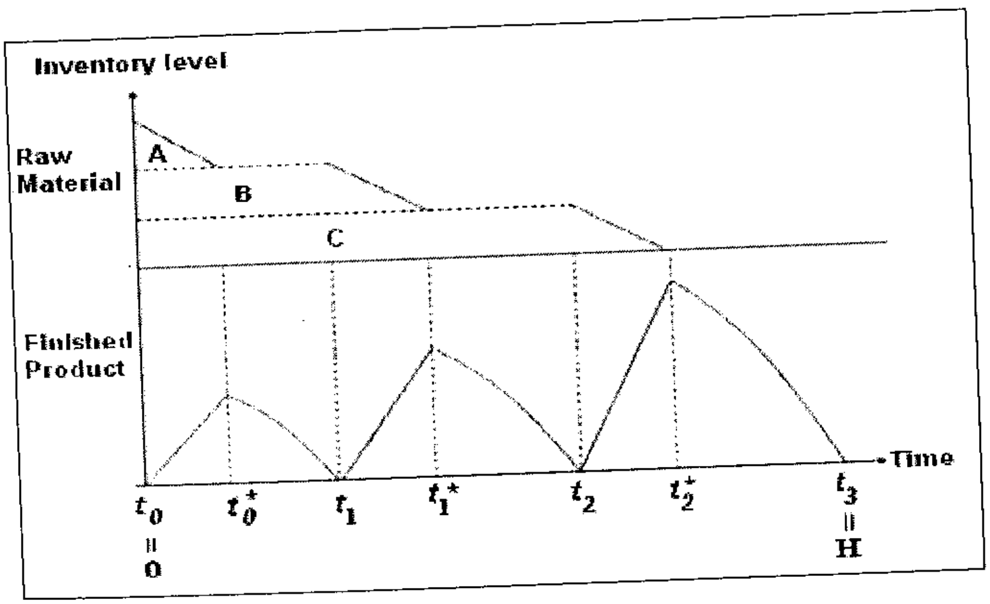

Figure 1. Plot of the inventory of raw material and finished product against time with $n = 3$

In order to obtain the total relevant cost of the model, we need to find the area below the graph. Let QR_{ij} be the quantity of raw material j needed for the whole production horizon. Then we have:

$$\sum_{i=0}^{n-1} QR_{ij} = r_j \int_0^H f(t)\,dt \qquad i = 0, 1, \ldots, n-1 \quad \textbf{(1)}$$

The total time-weighted stockholding for raw material is given by the areas A, B and C. It follows:

$$\left[\frac{1}{2P}\left(\int_{t_0}^{t_1} f(t)\;dt\right)^2\right] + \left[\frac{1}{2P}\left(\int_{t_1}^{t_2} f(t)\;dt\right)^2 + t_1\int_{t_1}^{t_2} f(t)\;dt\right] + \left[\frac{1}{2P}\left(\int_{t_2}^{H} f(t)\;dt\right)^2 + t_2\int_{t_2}^{H} f(t)\;dt\right] \quad \textbf{(2)}$$

Then for n-batch production cycle the total time weighted stockholding for raw material is

$$\frac{1}{2P}\sum_{i=0}^{n-1}\left[\int_{t_i}^{t_{i+1}} f(t)\;dt\right]^2 + \sum_{i=0}^{n-1} t_i\left[\int_{t_i}^{t_{i+1}} f(t)\;dt\right] \quad \textbf{(3)}$$

Similarly, the total time weighted stockholding for the finished product is (see Omar and Smith [2]):

$$\sum_{i=0}^{n-1}\left[\int_{t_i}^{t_{i+1}}\left(\int_{t}^{t_{i+1}} f(t)\,dt\right)dt\right] - \left[\int_{t_i}^{t_i^*}\left(\int_{t}^{t_{i+1}} f(t)\;dt\right) - \left(P(t - t_i) - \int_{t_i}^{t} f(t)\;dt\right)dt\right]$$

where the first term represents the area under the curve during production up-time and the second term is the area during production down-time.

Finally, the total cost for the production system for a given planning horizon is given by

$$TC = nc_p + h_p\left\{\sum_{i=0}^{n-1}\left[\int_{t_i}^{t_{i+1}}\left(\int_{t}^{t_{i+1}} f(t)dt\right)dt\right] - \left[\int_{t_i}^{t_i^*}\left(\int_{t}^{t_{i+1}} f(t)\;dt\right) - \left(P(t - t_i) - \int_{t_i}^{t} f(t)\;dt\right)dt\right]\right\} + c_1 + h_1 r_1\left[\frac{1}{2P}\sum_{i=0}^{n-1}\left(\int_{t_i}^{t_{i+1}} f(t)\;dt\right)^2 + \sum_{i=0}^{n-1} t_i\left(\int_{t_i}^{t_{i+1}} f(t)\;dt\right)\right] \quad \textbf{(4)}$$

For a linear increasing demand, $f(t) = a + bt$, we have

$$TC = nc_p + h_p \sum_{i=0}^{n-1} \frac{(t_{i+1} - t_i)^2}{2} \left\{ \left[a + \frac{b}{3}(2t_{i+1} + t_i) \right] - \frac{1}{P}\left[a + \frac{b}{2}(t_{i+1} + t_i) \right]^2 \right\}$$

$$+ c_1 + h_1 r_1 \left\{ \frac{1}{2P} \sum_{i=0}^{n-1} \left[a(t_{i+1} - t_i) + \frac{b}{2}\left(t_{i+1}^2 - t_i^2\right) \right]^2 \right. \qquad \textbf{(5)}$$

$$\left. + \sum_{i=0}^{n-1} t_i \left[a(t_{i+1} - t_i) + \frac{b}{2}\left(t_{i+1}^2 - t_i^2\right) \right] \right\}.$$

SOLUTION PROCEDURES

We find an optimal solution of the model by using an iterative optimal procedure by using the first principle of optimality.

For a fixed and given n, the necessary condition for the optimal t_i ($i = 1, 2, \ldots, n - 1$) are

$$\frac{\delta TC}{\delta t_i} = 0, \qquad \textbf{(6)}$$

with $t_i \geq 0$, and $t_i < t_{i+1} \leq H$.

Taking (6) into consideration then,

$$h_p \left\{ (t_i - t_{i-1}) \left\{ \left[a + \frac{b}{3}(2t_i + t_{i-1}) \right] - \frac{1}{P}\left[a + \frac{b}{2}(t_i + t_{i-1}) \right]^2 \right\} + \frac{(t_i - t_{i-1})^2}{2} \left\{ \frac{2b}{3} - \frac{b}{P}\left[a + \frac{b}{2}(t_i + t_{i-1}) \right] \right\} \right.$$

$$\left. - (t_{i+1} - t_i) \left\{ \left[a + \frac{b}{3}(2t_{i+1} + t_i) \right] - \frac{1}{P}\left[a + \frac{b}{2}(t_{i+1} + t_i) \right]^2 \right\} + \frac{(t_{i+1} - t_i)^2}{2} \left\{ \frac{b}{3} - \frac{b}{P}\left[a + \frac{b}{2}(t_{i+1} + t_i) \right] \right\} \right\}$$

$$+ h_1 \left\{ \frac{1}{P}\left[a(t_i - t_{i-1}) + \frac{b}{2}\left(t_i^2 - t_{i-1}^2\right) \right] [a + bt_i] + \frac{1}{P}\left[a(t_{i+1} - t_i) + \frac{b}{2}\left(t_{i+1}^2 - t_i^2\right) \right] [-a - bt_i] \right. \qquad \textbf{(7)}$$

$$\left. + [at_{i-1} + bt_i t_{i-1}] + \left[at_{i+1} - 2at_i + \frac{b}{2}t_{i+1}^2 - \frac{3b}{2}t_i^2 \right] \right\} = 0$$

$$= \frac{b}{2}t_{i+1}^2 + at_{i+1} + \frac{1}{(-a + P - bt_i)} \left[(a^2 - aP)(2t_i - t_{i-1}) + 3b\left(a - \frac{P}{2}\right)t_i^2 - \frac{ab}{2}t_{i-1}^2 + b^2\left(t_i^3 - \frac{t_i t_{i-1}}{2}\right) \right] = 0$$

where $i = 1, 2, \ldots, n - 1$.

Equation (7) can be simplified as:

$$at_{i+1} + b\,t_{i+1}^2 = F(t_{i-1}, \; t_i)$$

or,

$$t_{i+1} = -\frac{a}{b} + \frac{1}{b}\sqrt{a^2 + 2bF(t_{i-1}, \; t_i)} \qquad i = 1, 2, \ldots, n - 1 \qquad \textbf{(8)}$$

where,

$$F(t_{i-1}, \; t_i) = -\frac{1}{(-a + P - bt_i)} \left[(a^2 - aP)(2t_i - t_{i-1}) + 3b\left(a - \frac{P}{2}\right)t_i^2 - \frac{ab}{2}t_{i-1}^2 + b^2\left(t_i^3 - \frac{t_i t_{i-1}}{2}\right) \right]$$

Generally we are unable to show that $F(t) > 0$ for $0 \leq t_{i-1} \leq t_i < H$ ($i = 1, ..., n - 1$).

However in our numerical study, it is true for our cases, hence the squareroot portion of Equation (8) is always positive, therefore guaranteeing that t_{i+1} is always defined. Uniqueness of t_{i+1} is also guaranteed since the only other alternative will result in non-feasible negative value.

With $t_0 = 0$ and for a known value of t_1, the value of t_2 is easily deductible from Equation (8). We note that only the positive value of t_2 is taken into consideration. The obtainment of t_2 will in turn lead to t_3 by using Equation (8) recursively with the new t_2 as one of its argument. So, by continuing this process, all t_i's $i = 2, 3, ..., n$ can easily be found. By varying the value of t_1, we repeating this procedure until all t_i's are optimal or when $t_n = H$.

NUMERICAL EXAMPLE

To demonstrate the effectiveness of Model 3 we use this particular example. The parameter's values are:

$a = 100$	$b = 300$	$H = 5$
$P = 20000$	$c_p = 40$	$h_p = 2$
$h_1 = 0.1$		

By using the similar parameter, we compared the result of this model with lot-for-lot model (Model 1) developed by Omar and Smith [2] and lot-for-lot model with multiple installments of raw material (Model 2) developed by Omar and Supadi [3]. The minimum total cost with variation values of c_1 and h_1 for the case of $f(t) = 100 + 300t$ are shown in Table 1 and Table 2 respectively.

From Table 1, when $c_1 = 0.001, 0.003, ..., 0.03$ Model 2 gives the best minimum policy. However when $c_1 = 0.05, 0.07, ..., 70.00$, Models 1 and 2 give similar optimal policy. Thus when $c_1 \geq 0.05$ the multiple installments may not be cost effective, as a result the single installment is optimum in this case. However, when $c_1 = 70.00, 90.00, ..., 1000.00$ Model 3 gives the best policy due to the higher holding cost of raw material.

Figure 2 gives the minimum total cost against the ordering cost of raw material. The minimum total cost of Model 1 and 2 are more sensitive compared to Model 3. For example when the ordering cost of raw material increase from 100 to 400, the percentage increment of the minimum total cost for Model 1 and 2 is 79.65% and for Model 3 is 9.44%.

Table 2 represents the minimum total cost against the holding cost of raw material when $c_1 = 8$. As expected, Model 3 gives the best minimum total cost for very low raw material holding cost. When $h_1 > 0.01$ Model 1 become more superior than Model 3. Finally when $h_1 \geq 30$, Model 2 gives the best policy.

In Figure 3, the minimum total cost for all models against the holding cost of raw material are plotted. It shows that Model 3 is more sensitive compared to Model 1 and 2. Model 3 gives the best policy when the holding cost of raw material is not higher than 0.012.

Table 1. The minimum total cost of each model with different values of c_1

c_1	n*	TC* (MODEL 1)	n*	m*	TC* (MODEL 2)	n*	TC* (MODEL 3)
0.001	22	1747.7554	**22**	**10**	**1747.7554**	22	3077.2594
0.003	22	1747.7994	**22**	**6**	**1747.7994**	22	3077.2614
0.005	22	1747.8434	**22**	**5**	**1747.8434**	22	3077.2634
0.007	22	1747.8874	**22**	**4**	**1747.8874**	22	3077.2654
0.009	22	1747.9314	**22**	**3**	**1747.9314**	22	3077.2674
0.010	22	1747.9534	**22**	**3**	**1747.9534**	22	3077.2684
0.030	22	1748.3934	**22**	**2**	**1748.3934**	22	3077.2884
0.050	**22**	**1748.8334**	22	1	1748.8334	22	3077.3084
0.070	**22**	**1749.2734**	22	1	1749.2734	22	3077.3284
0.090	**22**	**1749.7134**	22	1	1749.7134	22	3077.3484
0.100	**22**	**1749.9334**	22	1	1749.9334	22	3077.3584
0.300	**22**	**1754.3334**	22	1	1754.3334	22	3077.5584
0.500	**22**	**1758.7334**	22	1	1758.7334	22	3077.7584
0.700	**22**	**1763.1334**	22	1	1763.1334	22	3077.9584
0.900	**22**	**1767.5334**	22	1	1767.5334	22	3078.1584
1.000	**22**	**1769.7334**	22	1	1769.7334	22	3078.2584
3.000	**21**	**1812.9457**	21	1	1812.9457	22	3080.2584
5.000	**21**	**1854.9457**	21	1	1854.9457	22	3082.2584
7.000	**20**	**1896.4708**	20	1	1896.4708	22	3084.2584
9.000	**20**	**1936.4708**	20	1	1936.4708	22	3086.2584
10.000	**20**	**1956.4708**	20	1	1956.4708	22	3087.2584
30.000	**17**	**2319.7256**	17	1	2319.7256	22	3107.2584
50.000	**15**	**2634.8325**	15	1	2634.8325	22	3127.2584
70.000	**13**	**2919.2172**	13	1	2919.2172	22	3147.2584
90.000	12	3177.8471	12	1	3177.8471	**22**	**3167.2584**
100.000	12	3297.8471	12	1	3297.8471	**22**	**3177.2584**
250.000	8	4790.4203	8	1	4790.4203	**22**	**3327.2584**
400.000	7	5924.4544	7	1	5924.4544	**22**	**3477.2584**
550.000	6	6891.1428	6	1	6891.1428	**22**	**3627.2584**
700.000	5	7775.6990	5	1	7775.6990	**22**	**3777.2584**
850.000	5	8525.6990	5	1	8525.6990	**22**	**3927.2584**
1000.000	5	9275.6990	5	1	9275.6990	**22**	**4077.2584**

Table 2. The minimum total cost of each model with different values of h_1

h_1	n*	TC*(MODEL 1)	n*	m*	TC*(MODEL 2)	n*	TC* (MODEL 3)
0.0005	20	1914.0627	20	1	1914.0627	**22**	**1760.1937**
0.0006	20	1914.0676	20	1	1914.0676	**22**	**1761.5254**
0.0007	20	1914.0676	20	1	1914.0676	**22**	**1762.8572**
0.0008	20	1914.0700	20	1	1914.0700	**22**	**1764.1889**
0.0009	20	1914.0724	20	1	1914.0724	**22**	**1765.5206**
0.0010	20	1914.0748	20	1	1914.0748	**22**	**1766.8523**
0.0030	20	1914.1234	20	1	1914.1234	**22**	**1793.4868**
0.0050	20	1914.1719	20	1	1914.1719	**22**	**1820.1213**
0.0070	20	1914.2204	20	1	1914.2204	**22**	**1846.7557**
0.0090	20	1914.2690	20	1	1914.2690	**22**	**1873.3902**
0.0100	20	1914.2933	20	1	1914.2933	**22**	**1886.7074**
0.0300	20	**1914.7770**	20	1	1914.7770	22	2153.0521
0.0500	20	**1915.2608**	20	1	1915.2608	22	2419.3967
0.0700	20	**1915.7446**	20	1	1915.7446	22	2685.7414
0.0900	20	**1916.2285**	20	1	1916.2285	22	2952.0860
0.1000	20	**1916.4708**	20	1	1769.7334	22	3085.2584
0.3000	20	**1921.3093**	20	1	1921.3093	20	5743.9430
0.5000	20	**1926.1454**	20	1	1926.1454	19	8397.0924
0.7000	20	**1930.9791**	20	1	1930.9791	18	11043.7628
0.9000	20	**1935.8103**	20	1	1935.8103	16	13682.6000
1.0000	20	**1938.2251**	20	1	1938.2251	16	14997.6364
3.0000	21	**1984.5257**	21	1	1984.5257	1	28001.3895
5.0000	21	**2030.2021**	21	1	2030.2021	1	28904.3895
7.0000	22	**2074.2755**	22	1	2074.2755	1	29807.5825
9.0000	22	**2117.5560**	22	1	2117.5560	1	30710.7755
10.0000	22	**2139.1473**	22	1	2139.1473	1	31162.3720
30.0000	26	2522.9026	**22**	**2**	**2422.6773**	1	40194.3021
50.0000	30	2851.6778	**24**	**2**	**2629.4058**	1	49226.2321
70.0000	33	3144.8154	**22**	**3**	**2776.6126**	1	58258.1621
90.0000	36	3412.0978	**23**	**3**	**2915.5261**	1	67290.0921
100.0000	37	3537.7196	**23**	**3**	**2982.9554**	1	71806.0571
250.0000	53	5056.9102	**23**	**5**	**3686.4742**	1	139545.5323
400.0000	65	6212.3940	**22**	**7**	**4193.1234**	1	207285.0074

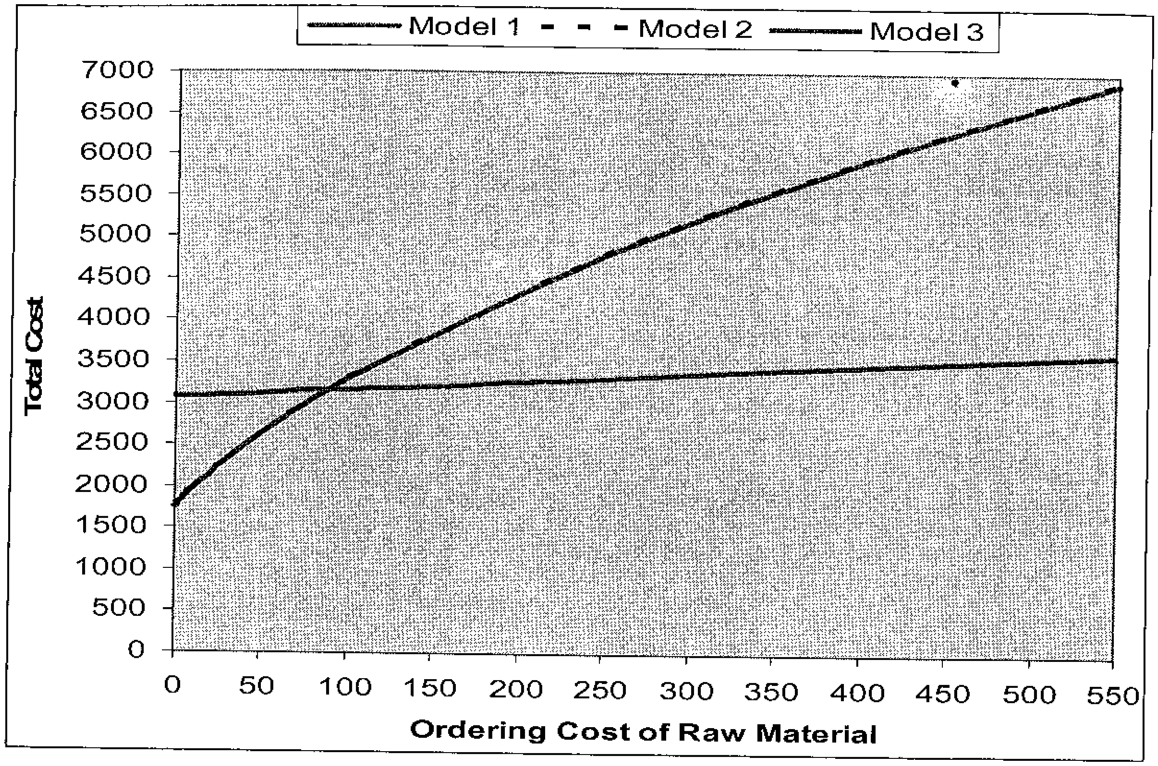

Figure 2. Plot of total cost against the ordering cost of raw material

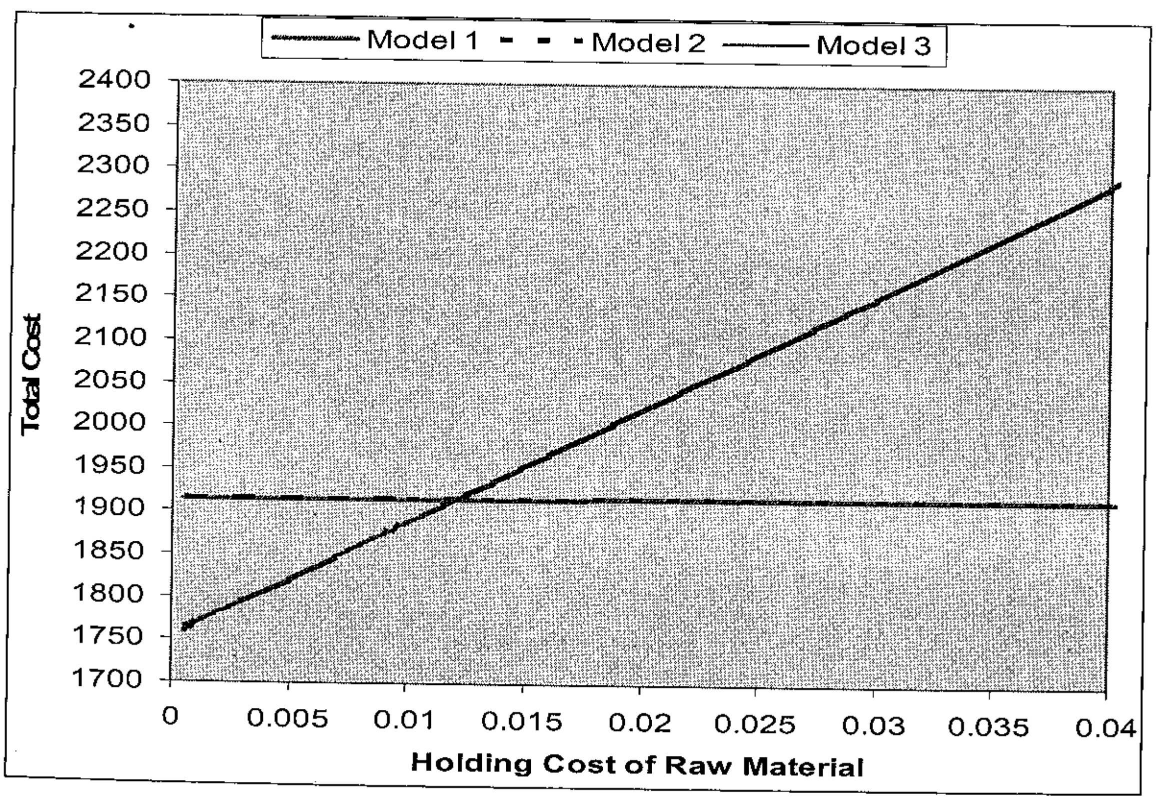

Figure 3. Plot of total cost against the holding cost of raw material

CONCLUSION

In this paper, an integrated production system for a single installments policy of raw material is developed. Our numerical results show that the proposed model is superior for a higher ordering cost of raw material or lower holding cost of raw material.

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