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The Applications of Metaheuristics in Inventory Routing Problems

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ABSTRACT Vendor-managed inventory (VMI) is one of the emerging solutions for improving the supply chain efficiency. It gives the vendors the responsibility to monitor and decide the inventory replenishments of their customers. VMI provides a win-win situation for both vendors and customers. The Inventory Routing Problem (IRP) is an important component of VMI policies. In this paper we consider a 1 *to many* distribution network consisting of a single depot and many geographically dispersed customers where each customer faces demands for a single item which arise at a deterministic and customer-specific rate. The problem addressed in this paper is based on a finite horizon, multi-period and single product where a fleet of capacitated vehicles transport items from the depot to meet the demand specified by the customers for each period. We propose a hybrid genetic algorithm based on allocation first, route second method to determine an optimal inventory and transportation policy that minimises the total cost. The computational results for un-split demand have been shown to be superior to those with split demand.

ABSTRAK Pengurusan Inventori oleh Vendor (Vendor-managed Inventory (VMI)) adalah satu pendekatan yang makin mendapat perhatian bagi meningkatkan kecekapan rangkaian rantaian pembekal. Dalam VMI, vendor dipertanggungjawabkan untuk memantau dan menambah (replenish) stok pelanggan yang berkenaan. VMI memberikan nilai tambah kepada kedua-dua vendor dan juga pelanggan. Masalah Inventory Routing (IRP) merupakan komponen penting bagi polisi VMI. Dalam kertas kerja ini kami mempertimbangkan satu rangkaian pengedaran berbentuk "1 *to many*" yang terdiri daripada satu depot dan beberapa pelanggan yang berselerak (geographically dispersed customers) yang mana setiap pelanggan mempunyai permintaan bagi satu produk yang berlaku secara berketentuan dan pada kadar yang bergantung kepada pelanggan. Masalah yang dipertimbangkan dalam kertas kerja ini adalah berasaskan ufuk finit, multi-period dan satu produk di mana satu rangkaian kenderaan (dengan kapasiti tertentu) menghantar barangan dari depot/gudang bagi memenuhi permintaan pelanggan dalam setiap period. Kami mengenengahkan satu algoritma genetik hibrid yang berasaskan kaedah peruntukan dahulu dan pembentukan jalan kemudian (allocation first, route second method) bagi menentukan polisi inventori dan pengangkutan yang optimum yang akan meminimumkan kos keseluruhan. Keputusan numerik bagi permintaan yang tidak dipecahkan adalah lebih baik dari permintaan yang dipecahkan kepada beberapa kenderaan.

(vendor-managed inventory, inventory routing problem, genetic algorithm)

INTRODUCTION

Vendor-managed inventory (VMI) is a business model where the vendors are provided certain information from their customers and become the central decision maker to replenish the customers' inventories by coordinating the demand and delivery [1]. It has been proven that VMI strategy can improve supply chain performance which consequently has led the VMI usage to grow over time [2]. By implementing VMI, both vendors and customers will have a win-win situation -- VMI helps the vendors to cut the distribution cost by coordinating better routes for deliveries and the customers can save on the inventory management and time as well. Another reason that draws a lot of attention towards VMI is the low cost of monitoring technology which allows the vendors to monitor the customers' inventory. For example, there are some information technologies such as Electronic Data Interchange (EDI) and Internet-based XML protocols where customers can share their sales and inventory information with vendors [3]. Using this information, the vendors can plan the deliveries and manage the inventories at the customers' stock-keeping facilities.

Initially, the VMI practitioners come from the petrochemical and industrial gas industry, but recently, many different industries such as the automotive industry, food and beverages industry, retail industry and electronics industry have adapted VMI into their systems [4].

The inventory routing problem (IRP) is an important component in VMI policies. It is an integration of the inventory management and vehicle routing which are also the components of supply chain management. IRP represents the main attributes which can be applied in VMI situations. The solution from the IRP problem can be used as the basic ground for logistics planning systems.

Among the first to study the inventory routing problem is Federgruen and Zipkin [5]. They approach the problem as a single day problem with a limited amount of inventory and the customers' demands are assumed to be a random variable. The task was to distribute the limited inventory among the customers while minimizing the transportation, inventory, and shortage costs in a given day. The problem was represented as a nonlinear integer program using a generalised Benders' decomposition approach. This approach has the attributes that for any assignment of customers to routes, the problem decomposes into a nonlinear inventory allocation problem which determines the inventory and shortage costs and a TSP for each vehicle considered which produces the transportation costs. However, not all customers will be visited every day as there are the inventory and shortage costs as well as the limited amount of inventory to be considered.

In 1986, Federgruen *et al.* [6] expand the idea for perishable product problem. The product in the system is divided into two classes. Old units is a class where the product will perish in the present period while fresh units are those with at least one period away from their perish date. The solution has been adopted from the earlier work of Federgruen and Zipkin [5] with a variation that the inventory allocation sub-problem accounts for two product classes.

Chien *et al.* [7] are among the first to simulate a multiple period planning model based on a single period approach. This is achieved by passing some information from one period to the next through inter-period inventory flow. In their problems, there is a central depot with many customers around it. The supply capacities of the depot and the demand of the customers are fixed. An integer program is modeled using a Lagrangean dual ascent method to handle the allocation of the limited inventory available at the plant to the customers, the customer to vehicle assignments, and the routing. The Lagrangean method is proposed in order to generate good upper bounds.

Fisher *et al.* [8] and Bell *et al.* [9] study a real-time problem of inventory routing at Air Products, an industrial gas producer. The objective of the work is to maximise the profit from product distributions over several days. The demand is given by upper and lower bounds on the amount to be delivered to each customer for every period in the planning horizon. By using the same approach as Chien *et al.* [7], Fisher *et al.* adopt Lagrangean dual ascent approach to solve the integer program.

Dror and Ball [10, 11] in their papers have considered the effect of the short-term over the long-term planning period. They proposed a mixed integer program where consequences of present decisions on later periods are accounted for using penalty and incentive factors. In this problem, the single period models are used as sub-problems. Dror and Levy [12] use the same approach to yield a weekly schedule and apply node and arc exchanges to reduce costs in the planning period.

Lee *et al.* [13] proposed a combination of Simulated Annealing and linear programs to solve an automotive part supply IRP which consists of multiple customers and an assembly plant. They address the problem as a finite horizon, multi-period, multi-supplier, single assembly plant part-supply network (many to 1 network). The objective of their study is to minimise the total transportation and inventory cost over the planning horizon. The problem is divided into two sub-problems, namely, vehicle routing and inventory control. The problem is modeled as a mixed integer programming and the Simulated Annealing is used to generate and evaluate alternative sets of vehicle routes while a linear program determine the optimum inventory levels for a given set of routes. In their work, Lee *et al.* also observed that the optimal solution is dominated by the transportation cost, regardless of the magnitude of the unit inventory carrying cost. In this model it is assumed that no backordering is allowed since any shortage of parts leads to excessively high costs at the assembly plant.

Kleywegt *et al.* [14] formulated a stochastic inventory routing problem with direct deliveries using Markov decision process. Since the Markov decision process can only solve very small problems, they propose an approximate dynamic programming approach in order to solve the problem which concentrates on single delivery per trip. In [15], they extend the approach to handle multiple deliveries per trip. An approximation is constructed based on a decomposition of the overall problem into subproblems.

Ribeiro and Lourenço [16] investigated IRP model for a multi-period problem with stochastic and deterministic demands. The objective is to design the set of routes and delivery quantities that minimise transportation cost while controlling inventory costs for two types of customers, namely, the vendor-managed inventory (VMI) customers and the customer managed inventory (CMI) customers. The former customers have a random demand and the distributor manages the stock at the customers' location. Meanwhile, the CMI type of customers has fixed demand and there are no inventory costs for the distributor. There are two solutions to be analysed from the integration of transportation and inventory, that is, the integrated solutions and the non-integrated solutions. The result shows that the inventory and transportation management in an integration model can yield a better performance. It is also worth noting that the inventory holding cost and the stock out cost are only incurred on the VMI customers. A more comprehensive review is given in [17].

In our work, we consider a 1 to *many* distribution network comprising of a single warehouse and a few scattering customers where each customer faces demands for a single item which arise at a deterministic and customer-specific rate. The problem addressed in this paper is based on a finite horizon, multi-period and single product where a fleet of capacitated vehicles transport items from the depot to meet the demand specified by the customers for each period. We propose a hybrid genetic algorithm based on allocation first, route second method to determine an optimal inventory and transportation policy that minimises the total cost.

In the following section we present the mathematical formulation of the problem. We then describe the Genetic Algorithm and the integration of the heuristics to solve the IRP. Following these, computational results and discussions for several randomly generated data and data taken from the literature are given. A conclusion is presented in the last section.

MATHEMATICAL FORMULATION

The mathematical formulation for the inventory routing problem with split demand is given as follows, which is a modification of [14]:

$$\text{Min} \quad V\sum_{i=0}^{m+1}\sum_{j=0}^{m+1}c_{ij}\left(\sum_{t=1}^{T}\sum_{k=1}^{J_t}x_{ijkt}\right)+\sum_{i=1}^{m}h_i\left(\sum_{t=0}^{T}s_{it}\right)+K\sum_{t=0}^{T}\sum_{k=1}^{J_t}y_{0kt} \tag{1a}$$

such that

$$0 \le a_{ikt} \le C\cdot y_{ikt}, \quad \forall i\in\{1,\dots,m\},\forall k,\forall t \tag{1b}$$

$$\sum_k a_{ikt} = a_{it}, \quad \forall i\in\{1,\dots,m\},\forall t \tag{1c}$$

$$\sum_i a_{ikt} \le C, \quad \forall k,\forall t \tag{1d}$$

$$\sum_j x_{ijkt} = \sum_j x_{j,i,k,t} = y_{i,k,t}, \quad \forall i\in\{1,\dots,m\},\forall k,\forall t \tag{1e}$$

$$\sum_{i,j\in U} x_{ijkt} \le |\mathrm{U}|-1, \quad U\subseteq\{s_1,s_2,\dots,s_m\} \tag{1f}$$

$$s_{it} = s_{it-1} + a_{it} - d_{it}, \quad \forall i\in\{1,\dots,m\},\forall t \tag{1g}$$

$$y_{ikt},x_{ijkt}\in\{0,1\}, \quad \forall i,j\in\{0,1,\dots,m+1\},\forall k,\forall t \tag{1h}$$

Input Parameters

- T Periods in the planning horizon
- C Capacity of the truck
- K The fixed cost per trip
- V Travel cost per unit distance
- d_{it} Demand for customer *i* in period *t*
- c_{ij} Travel distance between customer *i* to *j*
- h_i Unit inventory carrying cost for customer *i*
- J_t Upper bound on the number of trips needed in period *t*
- a_{ikt} Amount picked up by truck *k* from customer *i* in period *t*
- a_{it} Total amount to be picked up from customer *i* in period *t*
- s_{it} Inventory level of customer *i* at the end of period *t*
- s_i Customer *i*

0 – 1 Decision Variables

- x_{ijkt} = 1 if truck *k* visits customer *j* immediately after customer *i* in period *t*
- y_{ikt} = 1 if customer *i* is visited by truck *k* in period *t*

Let $\{0,1,\dots,m\}$ denote the set of customers, where 'supplier 0' is the warehouse/depot. For simplicity of terminology, a truck is assumed to perform one trip (route) in each period. However, this does not mean that the truck must not be used when it returns to the depot but will simply be given a different name so that 'truck' and 'trip' can be used interchangeably.

The objective function (1a) consists of fixed plus variable travel costs, and the inventory carrying cost. Meanwhile, constraint (1d) ensures that the truck capacity is not violated. Constraint (1f) serves as the sub-tour elimination constraint for each truck in each period. The inventory balance equation is given by constraint (1g). Constraint (1c) allows for the deliveries to be made using different vehicles (split demand). This ensures that the amount delivered (using different vehicles) in a given period is equivalent to the total deliveries in that particular period. We note that this constraint will be omitted for the un-split demand case.

In this study, the main objective is to minimise the total cost which comprise of inventory costs and the traveling cost (the traveling cost is assumed to be directly proportional to the total distance) for the whole planning horizon.

GENETIC ALGORITHMS FOR THE IRP

Genetic algorithm (GA) is a well-known, powerful searching tool which strikes a remarkable balance between exploration and exploitation of the search space. It has been used successfully in optimization problems such as wire routing, scheduling, transportation problems and traveling salesman problems [18]. To the best of our knowledge, this work is the first attempt to apply GA to solve IRP. However, in the course of writing this paper, some study has been carried out by Abdel Maguid and Dessouky [19] to investigate the effectiveness of modified genetic algorithm on the integrated inventory distribution problem, which is a different problem than the one presented in this paper.

We represent a genome or chromosome as a binary matrix of size $(N \times T-1)$ where N is the number of customers while T denotes the number of periods. We note that, without loss of generality, the initial inventory is assumed to be zero for all customers. A 1 at position (i, j) in the genome indicates that customer i will receive its delivery in period j. The amount to be delivered depends on whether there will be deliveries in the subsequent period or not. Since backordering is not allowed, the total delivery is the sum of all the demands in period $j, j+1,\ldots,k$ where the next delivery will be made in period k+1. As the initial inventory, s_{i0} for $i=1,2,\ldots,N$ is assumed to be zero, the values in the first column consist of all ones, and thus were ignored from the representation. However the algorithm can be adjusted accordingly if the initial inventory for customer i is given. After the total deliveries for each period has been determined, the routes are created using the sweep algorithm of Gillet and Miller [20].

We employ a two-dimensional uniform crossover where a binary mask of size $(N \times T-1)$ is generated randomly for each pair of parents. The position of the ones in the binary mask determines the values in the first parent that are transferred to the offspring and the zeros indicate that the values are taken from the second parent.

The algorithm can formally be stated as follows:

STEP 1: Generate a population of genomes randomly. Each genome is a binary matrix with the size $(N \times T-1)$. A 1 at position (i, j) in the genome indicates that customer i will receive its delivery in period j. Otherwise, there will be no deliveries.

STEP 2: Identify the customers that will receive deliveries in period j, $j=1,2,\ldots,T$. Construct the delivery matrix. Let $p_{ij}=\sum_{j=l}^{k} d_{ij}$ be the total deliveries in period j where the next delivery is only scheduled in period k.

STEP 3: For each period j, $j=1,2,\ldots,T$, arrange the customers around the depot. Sort the customers $\{s_1, s_2,\ldots,s_m\}$ in ascending order according to their angles. Let $s_{(i)}$ be the ith customer after the sort. Set $i=1$ and $k=1$. Open a route $R_k=\{\ \}$ and set $Q_k=0$, where Q_k is total delivery assigned to cluster k.

STEP 4 {Direct Deliveries}: If $p_{ij} \neq 0$ and $p_{ij} \geq C$, assign $s_{(i)}$ to route R_k, set $p_{ij}=p_{ij}-C$, $Q_k=C$, $k=k+1$ and open a new route, $R_k=\{\ \}$. Set $Q_k=0$.

STEP 5: If $p_{ij} \neq 0$ and $Q_k+p_{ij} \leq C$, assign $s_{(i)}$ to route R_k, set $Q_k=Q_k+p_{ij}$ and $a_{ik}=p_{ij}$. Otherwise set $a_{ik}=C-Q_k$ and $Q_k=C$. Set $k=k+1$ and open a new route $R_k=\{\ \}$, assign $s_{(i)}$ to R_k. Set $a_{ik}=p_{ij}-a_{ik-1}$ and $Q_k=a_{ik}$. If $i>m$, set

$k = 1$ and go to Step 5. Otherwise, set $i = i + 1$ and repeat Step 4.

STEP 6: Calculate the total objective which comprises of the total inventory costs and the total traveling distance for each genome.

STEP 7: Repeat Step 2 - Step 6 for all genomes.

RESULTS AND DISCUSSION

The algorithms were written in C++ and we used the GALIB to run the programs. We run the algorithm on four data sets. The first data set which comprises of 10 customers and 10 periods (labeled as S10T10) represents a small data set. This data set is generated randomly in 20x20 square and the demand is generated randomly in the interval of (0,30). The remaining data sets (S12T14, S20T21, S50T21) were taken from Lee at al.[14], which were developed for a many to one network IRP. The data is downloadable from http://www.mie.utoronto.ca/labs/ilr/IRP/. The number of generations, crossover rate and mutation rate are fixed at 300, 0.9 and 0.001 respectively for all the problems. We consider a population size of 50, 100, and 200 for each data set. All these values are chosen arbitrarily based on our limited experiments.

The programs were run on Pentium 4, 2.20 GHz with Microsoft Visual C++ 6.0. In this computation, the truck capacity for 10 customers and 10 periods problem is assumed to be 100 units while for the rest of the data, the truck capacity is 10 units.

Table 1 tabulates the results for split and un-split demand for each data set. Overall, it can be said that the split demand solution will decrease the number of truck compared to the un-split demand. However, the total cost for split demand is larger than the un-split demand. The table also shows that the total cost decreases when the population size increases. We observe that the algorithm produces superior results for problems with un-split demand. However, this is at the expense of significantly higher number of trucks. This may also be due to the fact that a fixed course associated with each truck is not incurred. If this cost is significant, the problem with split demand may produce better total objective function. We note that, to the best of our knowledge, there are no existing results from the literature that can be compared with. Lee *et al.* [14] solve for a different type of problem, where the vehicles are housed at the depot, which is different from the assembly plant.

In Figures 1, 2, and 3, we show the convergence graph for S10T10, S20T21 and S50T21 data sets for both split and un-split demands. The algorithm converges fairly fast for small data set.

It is observed that a population size of 200 converges to slightly better solutions as compared to population size of 50 and 100 in all data sets. The CPU time taken to run data set S50T21 for 200 genomes is roughly around 2700 seconds while for 100 genomes, the time taken is approximately around 1500 seconds.

CONCLUSION

It is important to consider the transportation and inventory costs simultaneously in the logistic planning functions as these two areas may lead to significant gains and more competitive distribution strategies, especially for VMI policies. In this paper, we present a solution for Inventory Routing Problem using the Genetic Algorithms approach on a finite horizon, multi-period, and a single product problem. The computational results show that, in the absence of fixed cost associated with each vehicle, the problems with un-split demand produce better total objective value as compared to the problem with split demand. This is however at the expense of a significantly higher number of vehicles.

Table 1. The results for the 4 data sets for split and un-split demands

Data Sets	Population Size											
	50 genomes				100 genomes				200 genomes			
	Split		Un-split		Split		Un-split		Split		Un-split	
S10T10	941.68[a]	0[b]	813.35	0	941.68	0	813.35	0	941.68	0	813.35	0
	941.68[c]	27[d]	813.35	28	941.68	27	813.35	28	941.68	27	813.35	28
S12T14	4807.9	645	4304.9	291	4703.6	666	4277.1	183	4564	579	4204.9	141
	4162.9	41	4013.9	43	4037.6	40	4094.1	42	3985	40	4063.9	41
S20T21	10628.6	1842	9898.3	1287	10474.6	1791	9774.7	1290	10115.5	1350	9697.8	1257
	8786.6	94	8611.3	99	8683.6	94	8484.7	101	8765.5	97	8440.8	98
S50T21	25004	4649	20603	3801	24232	4551	19734	3502	23535	4337	19726	3436
	20355	212	16802	238	19681	212	16232	232	19198	210	16290	236

a = Total Cost
b = Inventory Holding cost
c = Total Distance
d = Number of Trucks

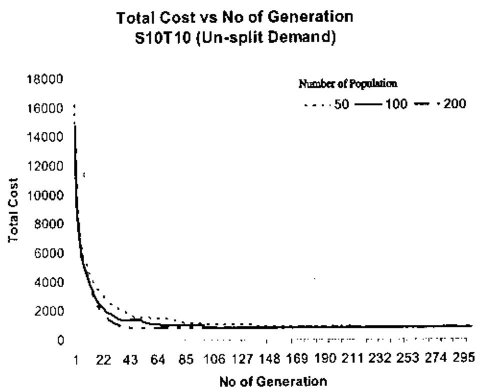

Figure 1(a). Graph for 10 customers and 10 periods (un-split demand)

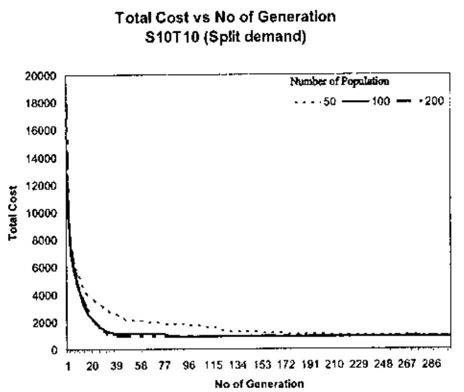

Figure 1(b). Graph for 10 customers and 10 periods (split demand)

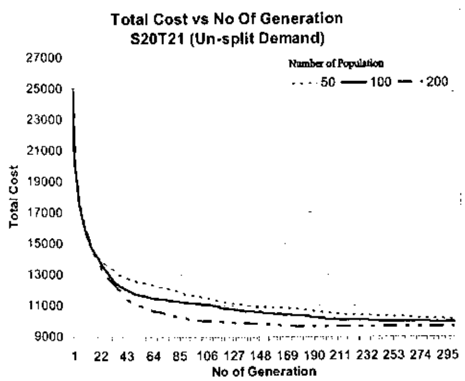

Figure 2(a). Graph for 20 customers and 21 periods (un-split demand)

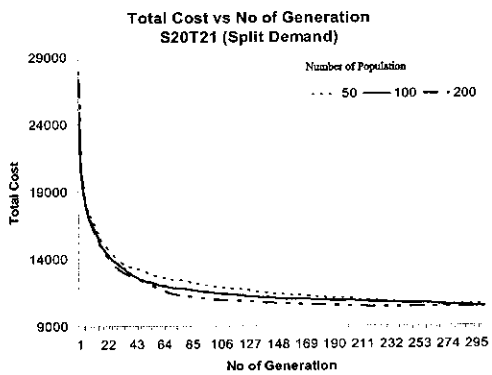

Figure 2(b). Graph for 20 customers and 21 periods (split demand)

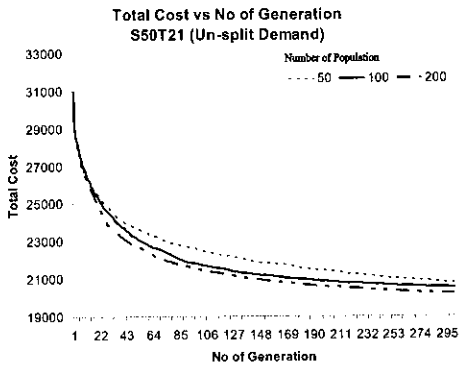

Figure 3(a). Graph for 50 customers and 21 periods (un-split demand)

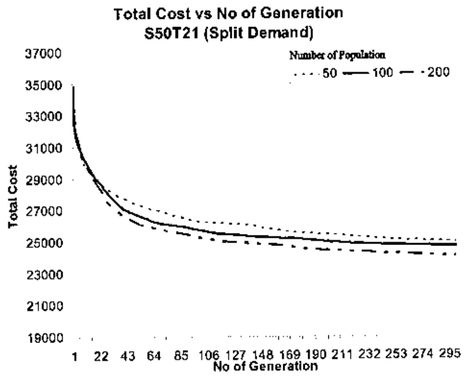

Figure 3(b): Graph for 50 customers and 21 periods (split demand)

REFERENCES

1. Wikipedia. Vendor managed inventory. http://en.wikipedia.org/wiki/Vendor_Managed_Inventory. Accessed Date: 7/6/2007.
2. Emigh, J. (1999). Vendor-managed inventory. *Computerworld* **33(34)** : 52.
3. Yao, Y., Evers, P.T. and Dresner, M.E. (2007). Supply chain integration in vendor-managed inventory. *Decision Support Systems* **43(2)**: 663-674.
4. Savelsbergh, M. The inventory routing problem. http://myweb.uiowa.edu/acampbll/inventoryrouting.htm. Accessed date: 14/03/2006.
5. Federgruen, A., and Zipkin, P. (1984). A combined vehicle routing and inventory allocation problem. *Operations Research* **32(5)** : 1019-1036.
6. Federgruen, A., Prastacos, G., and Zipkin, P. (1986). An allocation and distribution model for perishable products. *Operations Research* **34**:75-82.
7. Chien, T.W., Balakrishnan, A., and Wong, R.T. (1989). An integrated inventory allocation and vehicle routing problem. *Transportation Science* **23**: 1061-1076.
8. Fisher, M., Greenfield, A., Jaikumar, R. and Kedia, P. (1982). Real-time scheduling of a bulk delivery fleet: Practical application of Lagrangean relaxation. Technical Report, The Wharton School, University of Pennsylvania, Department of Decision Sciences.
9. Bell, W., Dalberto, L., Fisher, M., Greenfield, A., Jaikumar, R., Kedia, P., Mack, R. and Prutzman, P. (1983). Improving the distribution of industrial gases with an online computerized routing and scheduling optimizer. *Interfaces* **13(6)**:4-23.
10. Dror, M., Ball, M., and Golden, B. (1985). Computational comparison of algorithms for the inventory routing problem. *Annals of Operations Research* **4(1-4)**: 3-23.
11. Dror, M. and Ball, M. (1987). Inventory/routing: Reduction from an annual to a short period problem. *Naval Research Logistics Quarterly* **34(6)**: 891-905.
12. Dror, M. and Levy, L. (1986). Vehicle routing improvement algorithms: Comparison of a 'greedy' and a matching implementation for inventory routing. *Computers and Operations Research* **13(1)**: 33-45.
13. Lee,C-H., Bozer, Y.A. and White III C.C. (2003). A heuristic approach and properties of optimal solutions to the dynamic inventory routing problem. *Working Paper*, University of Toronto, Toronto, Ontario, Canada.

14. Kleywegt, A.J., Nori, V.S., and Savelsbergh, M.W.P. (2002). The stochastic inventory routing problem with direct deliveries. *Transportation Science* **36**:94-118.
15. Kleywegt, A.J., Nori, V.S., and Savelsbergh, M.W.P. (2004). Dynamic programming approximations for a stochastic inventory routing problem. *Transportation Science* **38**:42-70.
16. Ribeiro, R. and Laurenço, H.R. (2002). Inventory-routing Model, for a multi-period problem with stochastic and deterministic demand. *UPF Economics and Business Working paper.*
17. Moin, N.H. and Salhi, S. (2007). Inventory routing problems: a logistical overview. *Journal of the Operational Research Society* **58**:1185-1194.
18. Michalewicz, Z. (1994). *Genetic Algorithms+Data Structures=Evolution Programs.* New York: Springer-Verlag, pp 15.
19. Abdel Maguid, T.F. and Dessouky, M.M. (2006), A genetic algorithm approach to the integrated inventory-distribution problem. *International Journal of Production Research* **44**: 4445-4464.
20. Gillett, B.E. and Miller, L.R. (1974). A heuristic algorithm for the vehicle-dispatch problem, *Operations Research* **22**: 340-349.